USA
AMC 12/AHSME
1990

---

1 If $\frac{x/4}{2} = \frac{4}{x/2}$, then $x =$

(A) $\pm\frac{1}{2}$ (B) ± 1 (C) ± 2 (D) ± 4 (E) ± 8

2 $\left(\frac{1}{4}\right)^{-\frac{1}{4}} =$

(A) -16 (B) $-\sqrt{2}$ (C) $-\frac{1}{16}$ (D) $\frac{1}{256}$ (E) $\sqrt{2}$

3 The consecutive angles of a trapezoid form an arithmetic sequence. If the smallest angle is $75°$, then the largest angle is

(A) $95°$ (B) $100°$ (C) $105°$ (D) $110°$ (E) $115°$

4 Let ABCD be a parallelogram with $\angle ABC = 120°$, $AB = 16$, and $BC = 10$. Extend CD through D to E so that $DE = 4$. If BE intersects AD at F, then FD is closest to

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

5 Which of these numbers is largest?

(A) $\sqrt{\sqrt[3]{5 \cdot 6}}$ (B) $\sqrt{6\sqrt[3]{5}}$ (C) $\sqrt{5\sqrt[3]{6}}$ (D) $\sqrt[3]{5\sqrt{6}}$ (E) $\sqrt[3]{6\sqrt{5}}$

6 Points A and B are 5 units apart. How many lnes in a given plane containing A and B are 2 units from A and 3 units from B?

(A) 0 (B) 1 (C) 2 (D) 3 (E) more than 3

7 A triangle with integral sides has perimeter 8. The area of the triangle is

(A) $2\sqrt{2}$ (B) $\frac{16}{9}\sqrt{3}$ (C) $2\sqrt{3}$ (D) 4 (E) $4\sqrt{2}$

8 The number of real solutions to the equation $|x-2| + |x-3| = 1$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) more than 3

9 Each edge of a cube is colored either red or black.Every face of the cube has at least one black edge. The smallest possible number of black edges is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

10 An $11x11x11$ wooden cube is formed by gluing together 11^3 unit cubes. What is the greatest number of unit cubes that can be seen from a single point?

(A) 328 (B) 329 (C) 330 (D) 331 (E) 332

11 How man y positive integers less than 50 have an odd number of positive integer divisors?

(A) 3 (B) 5 (C) 7 (D) 9 (E) 11

12 Let f be the function defined by $f(x) = ax^2 - \sqrt{2}$ for some positive a. If $f(f(\sqrt{2})) = -\sqrt{2}$, then $a =$

(A) $\frac{2-\sqrt{2}}{2}$ (B) $\frac{1}{2}$ (C) $2 - \sqrt{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{2+\sqrt{2}}{2}$

13 If the following instructions are carried out by a computer, which of X will be printed because of instruction 5?

1. Start X at 3 and S at 0 2. Increase the value of X by 2. 3. Increase the value of S by the value of X. 4. If S is at least 10000, then go to instsruction 5; otherwise, go to instruction 2 and proceed from there. 5. Print the value of X. 6. Stop.

(A) 19 (B) 21 (C) 23 (D) 199 (E) 201

14 An acute isosceles triangle, ABC is inscribed in a circle. Through B and C, tangents to the circle are drawn, meeting at point D. If $\angle ABC = \angle ACB = 2\angle D$ and x is the radian measure of $\angle A$, then $x =$

[img]http://www.artofproblemsolving.com/Forum/album$_pic.php?pic_id = 766sid = f81d5c4216223342fe57ce$

(A) $\frac{3}{7}\pi$ (B) $\frac{4}{9}\pi$ (C) $\frac{5}{11}\pi$ (D) $\frac{6}{13}\pi$ (E) $\frac{7}{15}\pi$

15 Four whole numbers, when added three at a time, give the sums 180, 197, 208, and 222. What is the largest of the four numbers?

(A) 77 (B) 83 (C) 89 (D) 95 (E) cannot be determined

16 At one of George Washington's parties, each man shook hands with everyone except his spouse, and no handshakes took place between women. If 13 married couples attended, how many handshakes were there among these 26 people?

(A) 78 (B) 185 (C) 234 (D) 312 (E) 325

17 How many of the numbers, $100, 101, \ldots, 999$, have three different digits in increasing order or in decreasing order?

(A) 120 (B) 168 (C) 204 (D) 216 (E) 240

18 First a is chosen at random from the set $\{1, 2, 3, \ldots, 99, 100\}$, and then b is chosen at random from the same set. The probability that the integer $3^a + 7^b$ has units digit 8 is

(A) $\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $\frac{3}{16}$ (D) $\frac{1}{5}$ (E) $\frac{1}{4}$

19 For how many integers N between 1 and 1990 is the improper fraction $\frac{N^2+7}{N+4}$ not in lowest terms?

(A) 0 (B) 86 (C) 90 (D) 104 (E) 105

20 $ABCD$ is a quadrilateral with right angles at A and C. Points E and F are on AC, and DE and BF are perpendicular to AC. If $AE = 3$, $DE = 5$, and $CE = 7$, then $BF =$

(A) 3.6 (B) 4 (C) 4.2 (D) 4.5 (E) 5

21 Consider a pyramid $P-ABCD$ whose base $ABCD$ is a square and whose vertex P is equidistant from A, B, C, and D. If $AB = 1$ and $\angle APD = 2\theta$ then the volume of the pyramid is

(A) $\frac{\sin\theta}{6}$ (B) $\frac{\cot\theta}{6}$ (C) $\frac{1}{6\sin\theta}$ (D) $\frac{1-\sin 2\theta}{6}$ (E) $\frac{\sqrt{\cos 2\theta}}{6\sin\theta}$

22 If the six solutions of $x^6 = -64$ are written in the form $a + bi$, where a and b are real, then the product of those solutions with $a > 0$ is

(A) -2 (B) 0 (C) $2i$ (D) 4 (E) 16