USA
AMC 12/AHSME
1998

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1 Each of the sides of five congruent rectangles is labeled with an integer. In rectangle A, $w = 4, x = 1, y = 6, z = 9$. In rectangle B, $w = 1, x = 0, y = 3, z = 6$. In rectangle C, $w = 3, x = 8, y = 5, z = 2$. In rectangle D, $w = 7, x = 5, y = 4, z = 8$. In rectangle E, $w = 9, x = 2, y = 7, z = 0$. These five rectangles are placed, without rotating or reflecting, in position as below. Which of the rectangle is the top leftmost one?

(A) A (B) B (C) C (D) D (E) E

2 Letters A, B, C, and D represent four different digits from 0,1,2,3...9. If $(A+B)/(C+D)$ is an integer that is as large as possible, what is the value of $A+B$?

A) 13 B) 14 C) 15 D) 16 E) 17

3 If a, b, and c are digits for which

$$\begin{array}{cccc} & 7 & a & 2 \\ - & 4 & 8 & b \\ \hline & c & 7 & 3 \end{array}$$

then $a + b + c =$

(A) 14 (B) 15 (C) 16 (D) 17 (E) 18

4 Define $[a, b, c]$ to mean $\frac{a+b}{c}$, where $c \neq 0$. What is the value of $[[60, 30, 90], [2, 1, 3], [10, 5, 15]]$?

(A) 0 (B) 0.5 (C) 1 (D) 1.5 (E) 2

5 If $2^{1998} - 2^{1997} - 2^{1996} + 2^{1995} = k \cdot 2^{1995}$, what is the value of k?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

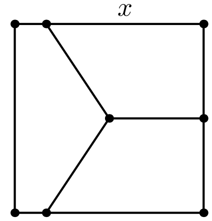
6 If 1998 is written as a product of two positive integers whose difference is as small as possible, then the difference is

(A) 8 (B) 15 (C) 17 (D) 47 (E) 93

7 If $N > 1$, then $\sqrt[3]{N\sqrt[3]{N\sqrt[3]{N}}} =$

(A) $N^{\frac{1}{27}}$ **(B)** $N^{\frac{1}{9}}$ **(C)** $N^{\frac{1}{3}}$ **(D)** $N^{\frac{13}{27}}$ **(E)** N

8 A square $ABCD$ with sides of length 1 is divided into two congruent trapezoids and a pentagon, which have equal areas, by joining the center of the square with points E, F, G where E is the midpoint of BC, F, G are on AB and CD, respectively, and they're positioned that $AF < FB, DG < GC$ and F is the directly opposite of G. If $FB = x$, the length of the longer parallel side of each trapezoid, find the value of x.

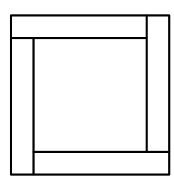


(A) $\frac{3}{5}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{5}{6}$ **(E)** $\frac{7}{8}$

9 A speaker talked for sixty minutes to a full auditorium. Twenty percent of the audience heard the entire talk and ten percent slept through the entire talk. Half of the remainder heard one third of the talk and the other half heard two thirds of the talk. What was the average number of minutes of the talk heard by members of the audience?

(A) 24 (B) 27 (C) 30 (D) 33 (E) 36

10 A large square is divided into a small square surrounded by four congruent rectangles as shown. The perimeter of each of the congruent rectangles is 14. What is the area of the large square?



(A) 49 **(B)** 64 **(C)** 100 **(D)** 121 **(E)** 196

11 Let R be a rectangle. How many circles in the plane of R have a diameter both of whose endpoints are vertices of R?

(A) 1 **(B)** 2 **(C)** 4 **(D)** 5 **(E)** 6

12 How many different prime numbers are factors of N if

$$\log_2(\log_3(\log_5(\log_7 N))) = 11?$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 7

13 Walter rolls four standard six-sided dice and finds that the product of the numbers on the upper face is 144. Which of the following could NOT be on the sum of the upper four faces?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

14 A parabola has vertex at (4,-5) and has two x-intercepts, one positive and one negative. If this parabola is the graph of $y = ax^2 + bx + c$, which of a, b, and c must be positive?

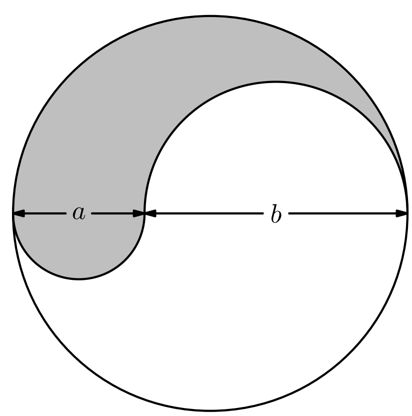
(A) Only a **(B)** Only b **(C)** Only c **(D)** Only a and b **(E)** None

15 A regular hexagon and an equilateral triangle have equal areas. What is the ratio of the length of a side of the triangle to the length of a side of the hexagon?

(A) $\sqrt{3}$ **(B)** 2 **(C)** $\sqrt{6}$ **(D)** 3 **(E)** 6

16 The figure shown is the union of a circle and two semicircles of diameters of a and b, all of whose centers are collinear. The ratio of the area of the shaded region to that of the unshaded region is

(A) $\sqrt{\frac{a}{b}}$ **(B)** $\frac{a}{b}$ **(C)** $\frac{a^2}{b^2}$ **(D)** $\frac{a+b}{2b}$ **(E)** $\frac{a^2+2ab}{b^2+2ab}$



17 Let $f(x)$ be a function with the two properties:

(a) for any two real numbers x and y, $f(x+y) = x + f(y)$, and (b) $f(0) = 2$

What is the value of $f(1998)$?

(A) 0 **(B)** 2 **(C)** 1996 **(D)** 1998 **(E)** 2000

18 A right circular cone of volume A, a right circular cylinder of volume M, and a sphere of volume C all have the same radius, and the common height of the cone and the cylinder is equal to the diameter of the sphere. Then

(A) $A - M + C = 0$ **(B)** $A + M = C$ **(C)** $2A = M + C$ **(D)** $A^2 - M^2 + C^2 = 0$ **(E)** $2A + 2M = 3C$

19 How many triangles have area 10 and vertices at $(-5, 0)$, $(5, 0)$, and $(5\cos\theta, 5\sin\theta)$ for some angle θ?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

20 Three cards, each with a positive integer written on it, are lying face-down on a table. Casey, Stacy, and Tracy are told that

(a) the numbers are all different, (b) they sum to 13, and (c) they are in increasing order, left to right

First, Casey looks at the number on the leftmost card and says, "I don't have enough information to determine the other two numbers." Then Tracy looks at the number on the rightmost card and says, "I don't have enough information to determine the other two numbers." Finally, Stacy looks at the number on the middle card and says, "I don't have enough information to determine the other two numbers." Assume that each perosn knows that the other two reason perfectly and hears their comments. What number is on the middle card?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** There is not enough information to determine the number.

21 In an h-meter race, Sunny is exactly d meters ahead of Windy when Sunny finishes the race. The next time they race, Sunny sportingly starts d meters behind Windy, who is at the starting line. Both runners run at the same constant speed as they did in the first race. How many meters ahead is Sunny when Sunny finishes the second race?

(A) $\frac{d}{h}$ **(B)** 0 **(C)** $\frac{d^2}{h}$ **(D)** $\frac{h^2}{d}$ **(E)** $\frac{d^2}{h-d}$

22 What is the value of the expression

$$\frac{1}{\log_2 100!} + \frac{1}{\log_3 100!} + \frac{1}{\log_4 100!} + \cdots + \frac{1}{\log_{100} 100!}?$$

(A) 0.01 **(B)** 0.1 **(C)** 1 **(D)** 2 **(E)** 10

23 The graphs of $x^2 + y^2 = 4 + 12x + 6y$ and $x^2 + y^2 = k + 4x + 12y$ intersect when k satisfies $a \le k \le b$, and for no other values of k. Find $b - a$.

(A) 5 **(B)** 68 **(C)** 104 **(D)** 140 **(E)** 144

24 Call a 7-digit telephone number $d_1d_2d_3 - d_4d_5d_6d_7$ *memorable* if the prefix sequence $d_1d_2d_3$ is exactly the same as either of the sequences $d_4d_5d_6$ or $d_5d_6d_7$ (possibly both). Assuming that each d_i can be any of the ten decimal digits $0, 1, 2, \ldots 9$, the number of different memorable telephone numbers is

(A) 19,810 **(B)** 19,910 **(C)** 19,990 **(D)** 20,000 **(E)** 20,100

25 A piece of graph paper is folded once so that $(0, 2)$ is matched with $(4, 0)$ and $(7, 3)$ is matched with (m, n). Find $m + n$.

(A) 6.7 **(B)** 6.8 **(C)** 6.9 **(D)** 7.0 **(E)** 8.0

26 In quadrilateral $ABCD$, it is given that $\angle A = 120^\circ$, angles B and D are right angles, $AB = 13$, and $AD = 46$. Then $AC =$

(A) 60 **(B)** 62 **(C)** 64 **(D)** 65 **(E)** 72

27 A $9 \times 9 \times 9$ cube is composed of twenty-seven $3 \times 3 \times 3$ cubes. The big cube is 'tunneled' as follows: First, the six $3 \times 3 \times 3$ cubes which make up the center of each face as well as the center of $3 \times 3 \times 3$ cube are removed. Second, each of the twenty remaining $3 \times 3 \times 3$ cubes is diminished in the same way. That is, the central facial unit cubes as well as each center cube are removed. The surface area of the final figure is

(A) 384 **(B)** 729 **(C)** 864 **(D)** 1024 **(E)** 1056

28 In triangle ABC, angle C is a right angle and $CB > CA$. Point D is located on $\overline{BC}$ so that angle CAD is twice angle DAB. If $AC/AD = 2/3$, then $CD/BD = m/n$, where m and n are relatively prime positive integers. Find $m + n$.

(A) 10 **(B)** 14 **(C)** 18 **(D)** 22 **(E)** 26

29 A point (x, y) in the plane is called a lattice point if both x and y are integers. The area of the largest square that contains exactly three lattice points in its interior is closest to

(A) 4.0 **(B)** 4.2 **(C)** 4.5 **(D)** 5.0 **(E)** 5.6

30 For each positive integer n, let:

$$a_n = \frac{(n+9)!}{(n-1)!}$$

If k is the smallest integer with the rightmost nonzero integer a_k is odd. The rightmost nonzero digit of a_k is

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9