# USA
## AMC 12/AHSME
2000

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1 In the year 2001, the United States will host the International Mathematical Olympiad. Let $I$, $M$, and $O$ be distinct positive integers such that the product $I \cdot M \cdot O = 2001$. What's the largest possible value of the sum $I + M + O$?

**(A)** 23 **(B)** 55 **(C)** 99 **(D)** 111 **(E)** 671

2 $2000(2000^{2000}) =$

**(A)** $2000^{2001}$ **(B)** $4000^{2000}$ **(C)** $2000^{4000}$ **(D)** $4,000,000^{2000}$ **(E)** $2000^{4,000,000}$

3 Each day, Jenny ate 20% of the jellybeans that were in her jar at the beginning of the day. At the end of the second day, 32 remained. How many jellybeans were in the jar originally?

**(A)** 40 **(B)** 50 **(C)** 55 **(D)** 60 **(E)** 75

4 The Fibonacci Sequence $1, 1, 2, 3, 5, 8, 13, 21, \ldots$ starts with two 1s and each term afterwards is the sum of its predecessors. Which one of the ten digits is the last to appear in thet units position of a number in the Fibonacci Sequence?

**(A)** 0 **(B)** 4 **(C)** 6 **(D)** 7 **(E)** 9

5 If $|x - 2| = p$, where $x < 2$, then $x - p =$

**(A)** $-2$ **(B)** 2 **(C)** $2 - 2p$ **(D)** $2p - 2$ **(E)** $|2p - 2|$

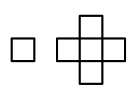
6 Two different prime numbers between 4 and 18 are chosen. When their sum is subtracted from their product, which of the following numbers could be obtained?

**(A)** 21 **(B)** 60 **(C)** 119 **(D)** 180 **(E)** 231

7 How many positive integers $b$ have the property that $\log_b 729$ is a positive integer?

**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

8 Figures 0, 1, 2, and 3 consist of 1, 5, 13, and 25 non-overlapping squares. If the pattern continued, how many non-overlapping squares would there be in figure 100?



**(A)** 10401 **(B)** 19801 **(C)** 20201 **(D)** 39801 **(E)** 40801

9 Mrs. Walter gave an exam in a mathematics class of five students. She entered the scores in random order into a spreadsheet, which recalculated the class average after each score was entered. Mrs. Walter noticed that after each score was entered, the average was always an

integer. The scores (listed in ascending order) were 71, 76, 80, 82, and 91. What was the last score Mrs. Walter entered?

**(A)** 71 **(B)** 76 **(C)** 80 **(D)** 82 **(E)** 91

10 The point $P = (1, 2, 3)$ is reflected in the $xy$-plane, then its image $Q$ is rotated by $180°$ about the $x$-axis to produce $R$, and finally, $R$ is translated by 5 units in the positive-$y$ direction to produce $S$. What are the coordinates of $S$?

**(A)** $(1, 7, -3)$ **(B)** $(-1, 7, -3)$ **(C)** $(-1, -2, 8)$ **(D)** $(-1, 3, 3)$ **(E)** $(1, 3, 3)$

11 Two non-zero real numbers, $a$ and $b$, satisfy $ab = a - b$. Which of the following is a possible value of $\frac{a}{b} + \frac{b}{a} - ab$?

**(A)** $-2$ **(B)** $-\frac{1}{2}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** 2

12 Let $A$, $M$, and $C$ be nonnegative integers such that $A + M + C = 12$. What is the maximum value of $A \cdot M \cdot C + A \cdot M + M \cdot C + C \cdot A$?

**(A)** 62 **(B)** 72 **(C)** 92 **(D)** 102 **(E)** 112

13 One morning each member of Angelas family drank an 8-ounce mixture of coffee with milk. The amounts of coffee and milk varied from cup to cup, but were never zero. Angela drank a quarter of the total amount of milk and a sixth of the total amount of coffee. How many people are in the family?

**(A)** 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

14 When the mean, median, and mode of the list

$$10, 2, 5, 2, 4, 2, x$$

are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real values of $x$?

**(A)** 3 **(B)** 6 **(C)** 9 **(D)** 17 **(E)** 20

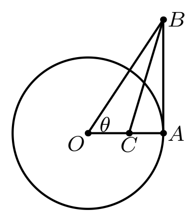
15 Let $f$ be a function for which $f(x/3) = x^2 + x + 1$. Find the sum of all values of $z$ for which $f(3z) = 7$.

**(A)** $-1/3$ **(B)** $-1/9$ **(C)** 0 **(D)** 5/9 **(E)** 5/3

16 A checkerboard of 13 rows and 17 columns has a number written in each square, beginning in the upper left corner, so that the rst row is numbered $1, 2, \ldots, 17$, the second row $18, 19, \ldots, 34$, and so on down the board. If the board is renumbered so that the left column, top to bottom, is $1, 2, \ldots, 13$, the second column $14, 15, \ldots, 26$ and so on across the board, some squares have the same numbers in both numbering systems. Find the sum of the numbers in these squares (under either system).

**(A)** 222 **(B)** 333 **(C)** 444 **(D)** 555 **(E)** 666

17 A circle centered at $O$ has radius 1 and contains the point $A$. Segment $AB$ is tangent to the circle at $A$ and $\angle AOB = \theta$. If point $C$ lies on $\overline{OA}$ and $\overline{BC}$ bisects $\angle ABO$, then $OC =$




**(A)** $\sec^2\theta - \tan\theta$ **(B)** $\frac{1}{2}$ **(C)** $\frac{\cos^2\theta}{1+\sin\theta}$ **(D)** $\frac{1}{1+\sin\theta}$ **(E)** $\frac{\sin\theta}{\cos^2\theta}$

18 In year $N$, the $300^{\text{th}}$ day of the year is a Tuesday. In year $N+1$, the $200^{\text{th}}$ day of the year is also a Tuesday. On what day of the week did the $100^{\text{th}}$ day of year $N-1$ occur?

**(A)** Thursday **(B)** Friday **(C)** Saturday **(D)** Sunday **(E)** Monday

19 In triangle $ABC$, $AB = 13$, $BC = 14$, and $AC = 15$. Let $D$ denote the midpoint of $\overline{BC}$ and let $E$ denote the intersection of $\overline{BC}$ with the bisector of angle $BAC$. Which of the following is closest to the area of the triangle $ADE$?

**(A)** 2 **(B)** 2.5 **(C)** 3 **(D)** 3.5 **(E)** 4

20 If $x$, $y$, and $z$ are positive numbers satisfying

$x + 1/y = 4$, $y + 1/z = 1$, and $z + 1/x = 7/3$

then $xyz =$

**(A)** 2/3 **(B)** 1 **(C)** 4/3 **(D)** 2 **(E)** 7/3

21 Through a point on the hypotenuse of a right triangle, lines are drawn parallel to the legs of the triangle so that the triangle is divided into a square and two smaller right triangles. The area of one of the two small right triangles is $m$ times the area of the square. The ratio of the area of the other small right triangle to the area of the square is

**(A)** $\frac{1}{2m+1}$ **(B)** $m$ **(C)** $1-m$ **(D)** $\frac{1}{4m}$ **(E)** $\frac{1}{8m^2}$

22 The graph below shows a portion of the curve defined by the quartic polynomial $P(x) = x^4 + ax^3 + bx^2 + cx + d$. Which of the following is the smallest?

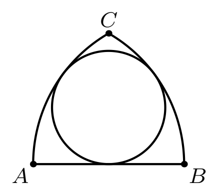
**(A)** $P(-1)$ **(B)** The product of the zeros of $P$ **(C)** The product of the non - real zeros of $P$ **(D)** The sum of the coefficients of $P$ **(E)** The sum of the real zeros of $P$

[img]13164[/img]

23 Professor Gamble buys a lottery ticket, which requires that he pick six different integers from 1 through 46, inclusive. He chooses his numbers so that the sum of the base-ten logarithms of his six numbers is an integer. It so happens that the integers on the winning ticket have the same property the sum of the base-ten logarithms is an integer. What is the probability that Professor Gamble holds the winning ticket?

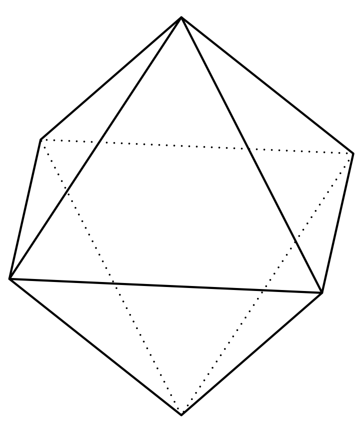
**(A)** 1/5 **(B)** 1/4 **(C)** 1/3 **(D)** 1/2 **(E)** 1

24 If circular arcs $AC$ and $BC$ have centers at $B$ and $A$, respectively, then there exists a circle tangent to both $\overset{\frown}{AC}$ and $\overset{\frown}{BC}$, and to $\overline{AB}$. If the length of $\overset{\frown}{BC}$ is 12, then the circumference of the circle is




**(A)** 24 **(B)** 25 **(C)** 26 **(D)** 27 **(E)** 28

25 Eight congruent equilateral triangles, each of a different color, are used to construct a regular octahedron. How many distinguishable ways are there to construct the octahedron? (Two colored octahedrons are distinguishable if neither can be rotated to look just like the other.)



**(A)** 210 **(B)** 560 **(C)** 840 **(D)** 1260 **(E)** 1680