USA
AMC 12/AHSME
2002

A

1 Compute the sum of all the roots of $(2x+3)(x-4)+(2x+3)(x-6)=0$.

(A) 7/2 **(B)** 4 **(C)** 5 **(D)** 7 **(E)** 13

2 Cindy was asked by her teacher to subtract 3 from a certain number and then divide the result by 9. Instead, she subtracted 9 and then divided the result by 3, giving an answer of 43. What would her answer have been had she worked the problem correctly?

(A) 15 **(B)** 34 **(C)** 43 **(D)** 51 **(E)** 138

3 According to the standard convention for exponentiation,

$$2^{2^{2^{2}}} = 2^{\left(2^{\left(2^2\right)}\right)} = 2^{16} = 65{,}536.$$

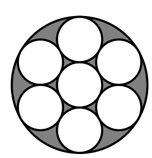
If the order in which the exponentiations are performed is changed, how many other values are possible?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4 Find the degree measure of an angle whose complement is 25% of its supplement.

(A) 48 **(B)** 60 **(C)** 75 **(D)** 120 **(E)** 150

5 Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. Find the area of the shaded region.



(A) π **(B)** 1.5π **(C)** 2π **(D)** 3π **(E)** 3.5π

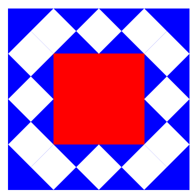
6 For how many positive integers m does there exist at least one positive integer n such that $m \cdot n \le m + n$?

(A) 4 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** infinitely many

7 If an arc of 45° on circle A has the same length as an arc of 30° on circle B, then the ratio of the area of circle A to the area of circle B is

(A) $\frac{4}{9}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{5}{6}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{9}{4}$

8 Betsy designed a flag using blue triangles, small white squares, and a red center square, as shown. Let B be the total area of the blue triangles, W the total area of the white squares, and R the area of the red square. Which of the following is correct?



(A) $B = W$ **(B)** $W = R$ **(C)** $B = R$ **(D)** $3B = 2R$ **(E)** $2R = W$

9 Jamal wants to store 30 computer files on floppy disks, each of which has a capacity of 1.44 megabytes (MB). Three of his files require 0.8 MB of memory each, 12 more require 0.7 MB each, and the remaining 15 require 0.4 MB each. No file can be split between floppy disks. What is the minimal number of floppy disks that will hold all the files?

(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

10 Sarah pours four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. She then transfers half the coffee from the first cup to the second and, after stirring thoroughly, transfers half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?

(A) 1/4 **(B)** 1/3 **(C)** 3/8 **(D)** 2/5 **(E)** 1/2

11 Mr. Earl E. Bird leaves his house for work at exactly 8:00 A.M. every morning. When he averages 40 miles per hour, he arrives at his workplace three minutes late. When he averages 60 miles per hour, he arrives three minutes early. At what average speed, in miles per hour, should Mr. Bird drive to arrive at his workplace precisely on time?

(A) 45 **(B)** 48 **(C)** 50 **(D)** 55 **(E)** 58

12 Both roots of the quadratic equation $x^2 - 63x + k = 0$ are prime numbers. The number of possible values of k is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E) more than four**

13 Two different numbers a and b each differ from their reciprocals by 1. What is $a + b$?

(A) 1 **(B)** 2 **(C)** $\sqrt{5}$ **(D)** $\sqrt{6}$ **(E)** 3

14 For all positive integers n, let $f(n) = \log_{2002} n^2$. Let

$$N = f(11) + f(13) + f(14)$$

Which of the following relations is true?

(A) $N < 1$ **(B)** $N = 1$ **(C)** $1 < N < 2$ **(D)** $N = 2$ **(E)** $N > 2$

15 The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element of this collection is

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

16 Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$ and Sergio randomly selects a number from the set $\{1, 2, ..., 10\}$. The probability that Sergio's number is larger than the sum of the two numbers chosen by Tina is

(A) 2/5 **(B)** 9/20 **(C)** 1/2 **(D)** 11/20 **(E)** 24/25

17 Several sets of prime numbers, such as $\{7, 83, 421, 659\}$ use each of the nine nonzero digits exactly once. What is the smallest possible sum such a set of primes could have?

(A) 193 **(B)** 207 **(C)** 225 **(D)** 252 **(E)** 447

18 Let C_1 and C_2 be circles defined by

$$(x - 10)^2 + y^2 = 36$$

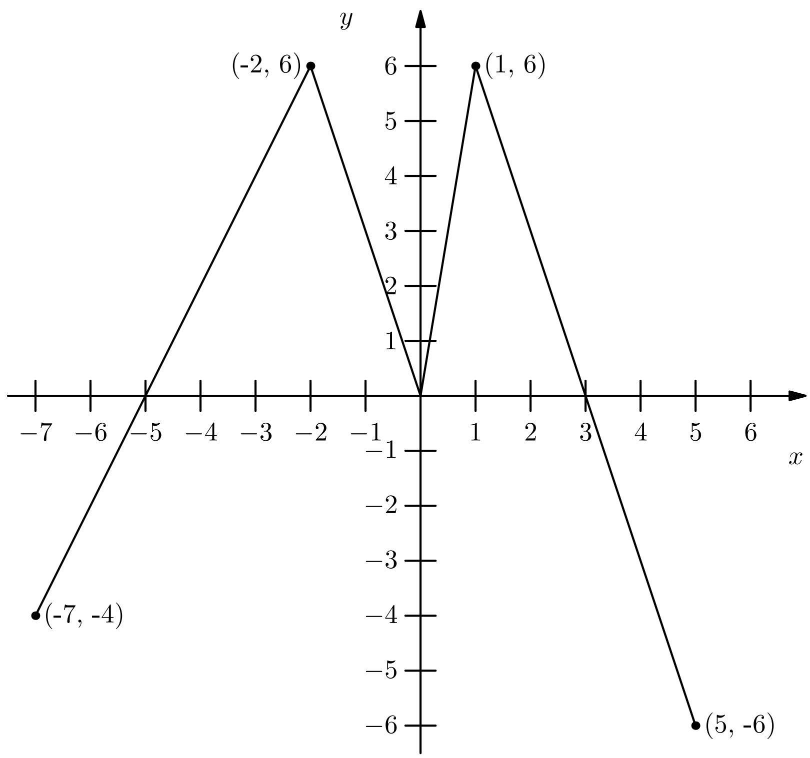
and

$$(x + 15)^2 + y^2 = 81,$$

respectively. What is the length of the shortest line segment $\overline{PQ}$ that is tangent to C_1 at P and to C_2 at Q?

(A) 15 **(B)** 18 **(C)** 20 **(D)** 21 **(E)** 24

19 The graph of the function f is shown below. How many solutions does the equation $f(f(x)) = 6$ have?



(A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

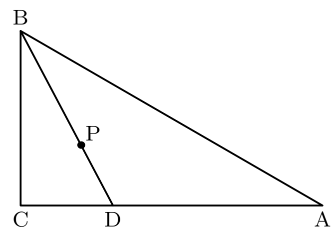
20 Suppose that a and b are digits, not both nine and not both zero, and the repeating decimal $0.\overline{ab}$ is expressed as a fraction in lowest terms. How many different denominators are possible?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 8 **(E)** 9

21 Consider the sequence of numbers: $4, 7, 1, 8, 9, 7, 6, \ldots$. For $n > 2$, the nth term of the sequence is the units digit of the sum of the two previous terms. Let S_n denote the sum of the first n terms of this sequence. The smallest value of n for which $S_n > 10,000$ is:

(A) 1992 **(B)** 1999 **(C)** 2001 **(D)** 2002 **(E)** 2004

22 Triangle ABC is a right triangle with $\angle ACB$ as its right angle, $m\angle ABC = 60°$, and $AB = 10$. Let P be randomly chosen inside $\triangle ABC$, and extend $\overline{BP}$ to meet $\overline{AC}$ at D. What is the probability that $BD > 5\sqrt{2}$?



(A) $\frac{2-\sqrt{2}}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3-\sqrt{3}}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{5-\sqrt{5}}{5}$

23 In triangle ABC, side AC and the perpendicular bisector of BC meet in point D, and BD bisects $\angle ABC$. If $AD = 9$ and $DC = 7$, what is the area of triangle ABD?

(A) 14 **(B)** 21 **(C)** 28 **(D)** $14\sqrt{5}$ **(E)** $28\sqrt{5}$

24 Find the number of ordered pairs of real numbers (a, b) such that $(a + bi)^{2002} = a - bi$.

(A) 1001 **(B)** 1002 **(C)** 2001 **(D)** 2002 **(E)** 2004

25 The nonzero coefficients of a polynomial P with real coefficients are all replaced by their mean to form a polynomial Q. Which of the following could be a graph of $y = P(x)$ and $y = Q(x)$ over the interval $-4 \le x \le 4$?

(A)[img]13457[/img]

(B)[img]13458[/img]

(C)[img]13459[/img]

(D)[img]13460[/img]

(E)[img]13461[/img]

USA
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B

1. The arithmetic mean of the nine numbers in the set $\{9, 99, 999, 9999, ..., 999999999\}$ is a 9-digit number M, all of whose digits are distinct. The number M does not contain the digit

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

2. What is the value of
$$(3x - 2)(4x + 1) - (3x - 2)4x + 1$$
when $x = 4$?

   (A) 0 **(B)** 1 **(C)** 10 **(D)** 11 **(E)** 12

3. For how many positive integers n is $n^2 - 3n + 2$ a prime number?

   (A) none **(B)** one **(C)** two **(D)** more than two, but finitely many **(E)** infinitely many

4. Let n be a positive integer such that $\frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{n}$ is an integer. Which of the following statements is **not** true?

   (A) 2 divides n **(B)** 3 divides n **(C)** 6 divides n **(D)** 7 divides n **(E)** $n > 84$

5. Let v, w, x, y, and z be the degree measures of the five angles of a pentagon. Suppose $v < w < x < y < z$ and v, w, x, y, and z form an arithmetic sequence. Find the value of x.

   (A) 72 **(B)** 84 **(C)** 90 **(D)** 108 **(E)** 120

6. Suppose that a and b are are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has positive solutions a and b. Then the pair (a, b) is

   (A) $(-2, 1)$ **(B)** $(-1, 2)$ **(C)** $(1, -2)$ **(D)** $(2, -1)$ **(E)** $(4, 4)$

7. The product of three consecutive positive integers is 8 times their sum. What is the sum of their squares?

   (A) 50 **(B)** 77 **(C)** 110 **(D)** 149 **(E)** 194

8. Suppose July of year N has five Mondays. Which of the following must occur five times in August of year N? (Note: Both months have 31 days.)

   (A) Monday **(B)** Tuesday **(C)** Wednesday **(D)** Thursday **(E)** Friday

9. If a, b, c, and d are positive real numbers such that a, b, c, d form an increasing arithmetic sequence and a, b, d form a geometric sequence, then $\frac{a}{d}$ is

   (A) $\frac{1}{12}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

10. How many different integers can be expressed as the sum of three distinct members of the set $\{1, 4, 7, 10, 13, 16, 19\}$?

(A) 13 **(B)** 16 **(C)** 24 **(D)** 30 **(E)** 35

11. The positive integers A, B, $A - B$, and $A + B$ are all prime numbers. The sum of these four primes is

(A) even **(B)** divisible by 3 **(C)** divisible by 5 **(D)** divisible by 7
(E) prime

12. For how many integers n is $\frac{n}{20-n}$ the square of an integer?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 10

13. The sum of 18 consecutive positive integers is a perfect square. The smallest possible value of this sum is

(A) 169 **(B)** 225 **(C)** 289 **(D)** 361 **(E)** 441

14. Four distinct circles are drawn in a plane. What is the maximum number of points where at least two of the circles intersect?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** 16

15. How many four-digit numbers N have the property that the three-digit number obtained by removing the leftmost digit is one ninth of N?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

16. Juan rolls a fair regular octahedral die marked with the numbers 1 through 8. Then Amal rolls a fair six-sided die. What is the probability that the product of the two rolls is a multiple of 3?

(A) $\frac{1}{12}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{7}{12}$ **(E)** $\frac{2}{3}$

17. Andys lawn has twice as much area as Beths lawn and three times as much area as Carlos lawn. Carlos lawn mower cuts half as fast as Beths mower and one third as fast as Andys mower. If they all start to mow their lawns at the same time, who will nish rst?

(A) Andy **(B)** Beth **(C)** Carlos **(D)** Andy and Carlos tie for first.
(E) All three tie.

18. A point P is randomly selected from the rectangular region with vertices $(0, 0)$, $(2, 0)$, $(2, 1)$, $(0, 1)$. What is the probability that P is closer to the origin than it is to the point $(3, 1)$?

(A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{4}{5}$ **(E)** 1

19 If a, b, and c are positive real numbers such that $a(b+c) = 152$, $b(c+a) = 162$, and $c(a+b) = 170$, then abc is

(A) 672 **(B)** 688 **(C)** 704 **(D)** 720 **(E)** 750

20 Let $\triangle XOY$ be a right-angled triangle with $m\angle XOY = 90^\circ$. Let M and N be the midpoints of legs OX and OY, respectively. Given that $XN = 19$ and $YM = 22$, find XY.

(A) 24 **(B)** 26 **(C)** 28 **(D)** 30 **(E)** 32

21 For all positive integers n less than 2002, let

$$a_n = \begin{cases} 11 & \text{if } n \text{ is divisible by 13 and 14} \\ 13 & \text{if } n \text{ is divisible by 11 and 14} \\ 14 & \text{if } n \text{ is divisible by 11 and 13} \\ 0 & \text{otherwise} \end{cases}$$

Calculate $\sum_{n=1}^{2001} a_n$.

(A) 448 **(B)** 486 **(C)** 1560 **(D)** 2001 **(E)** 2002

22 For all integers n greater than 1, define $a_n = \frac{1}{\log_n 2002}$. Let $b = a_2 + a_3 + a_4 + a_5$ and $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then $b - c$ equals

(A) -2 **(B)** -1 **(C)** $\frac{1}{2002}$ **(D)** $\frac{1}{1001}$ **(E)** $\frac{1}{2}$

23 In $\triangle ABC$, we have $AB = 1$ and $AC = 2$. Side BC and the median from A to BC have the same length. What is BC?

(A) $\frac{1+\sqrt{2}}{2}$ **(B)** $\frac{1+\sqrt{3}}{2}$ **(C)** $\sqrt{2}$ **(D)** $\frac{3}{2}$ **(E)** $\sqrt{3}$

24 A convex quadrilateral $ABCD$ with area 2002 contains a point P in its interior such that $PA = 24$, $PB = 32$, $PC = 28$, and $PD = 45$. FInd the perimeter of $ABCD$.

(A) $4\sqrt{2002}$ **(B)** $2\sqrt{8465}$ **(C)** $2\left(48+\sqrt{2002}\right)$ **(D)** $2\sqrt{8633}$ **(E)** $4\left(36+\sqrt{113}\right)$

25 Let $f(x) = x^2 + 6x + 1$, and let R denote the set of points (x, y) in the coordinate plane such that

$$f(x) + f(y) \le 0 \text{ and } f(x) - f(y) \le 0$$

The area of R is closest to

(A) 21 **(B)** 22 **(C)** 23 **(D)** 24 **(E)** 25