USA
AMC 12/AHSME
2003

A

1 What is the difference between the sum of the first 2003 even counting numbers and the sum of the first 2003 odd counting numbers?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 2003 **(E)** 4006

2 Members of the Rockham Soccer League buy socks and T-shirts. Socks cost \$4 per pair and each T-shirt costs \$5 more than a pair of socks. Each member needs one pair of socks and a shirt for home games and another pair of socks and a shirt for away games. If the total cost is \$2366, how many members are in the League?

(A) 77 **(B)** 91 **(C)** 143 **(D)** 182 **(E)** 286

3 A solid box is 15 cm by 10 cm by 8 cm. A new solid is formed by removing a cube 3 cm on a side from each corner of this box. What percent of the original volume is removed?

(A) 4.5 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 24

4 It takes Mary 30 minutes to walk uphill 1 km from her home to school, but it takes her only 10 minutes to walk from school to home along the same route. What is her average speed, in km/hr, for the round trip?

(A) 3 **(B)** 3.125 **(C)** 3.5 **(D)** 4 **(E)** 4.5

5 The sum of the two 5-digit numbers $AMC10$ and $AMC12$ is 123422. What is $A + M + C$?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

6 Define $x\heartsuit y$ to be $|x - y|$ for all real numbers x and y. Which of the following statements is **not** true?

(A) $x\heartsuit y = y\heartsuit x$ for all x and y
(B) $2(x\heartsuit y) = (2x)\heartsuit(2y)$ for all x and y
(C) $x\heartsuit 0 = x$ for all x
(D) $x\heartsuit x = 0$ for all x
(E) $x\heartsuit y > 0$ if $x \neq y$

7 How many non-congruent triangles with perimeter 7 have integer side lengths?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

8 What is the probability that a randomly drawn positive factor of 60 is less than 7?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

9 A set S of points in the xy-plane is symmetric about the origin, both coordinate axes, and the line $y = x$. If $(2, 3)$ is in S, what is the smallest number of points in S?

(A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

10 Al, Bert, and Carl are the winners of a school drawing for a pile of Halloween candy, which they are to divide in a ratio of $3 : 2 : 1$, respectively. Due to some confusion they come at different times to claim their prizes, and each assumes he is the rst to arrive. If each takes what he believes to be his correct share of candy, what fraction of the candy goes unclaimed?

(A) $\frac{1}{18}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{2}{9}$ **(D)** $\frac{5}{18}$ **(E)** $\frac{5}{12}$

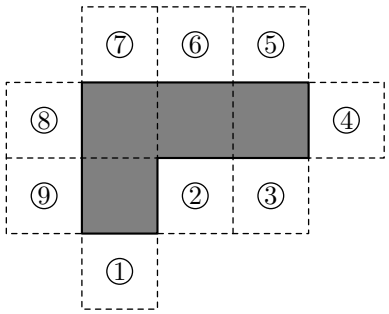
11 A square and an equilateral triangle have the same perimeter. Let A be the area of the circle circumscribed about the square and B be the area of the circle circumscribed about the triangle. Find A/B.

(A) $\frac{9}{16}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{27}{32}$ **(D)** $\frac{3\sqrt{6}}{8}$ **(E)** 1

12 Sally has ve red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card divides evenly into the number on each neighboring blue card. What is the sum of the numbers on the middle three cards?

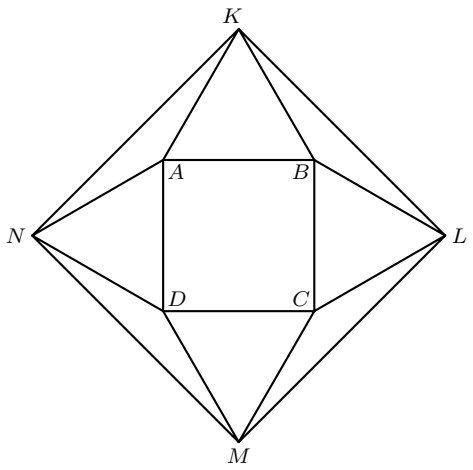
(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

13 The polygon enclosed by the solid lines in the gure consists of 4 congruent squares joined edge-to-edge. One more congruent square is attached to an edge at one of the nine positions indicated. How many of the nine resulting polygons can be folded to form a cube with one face missing?



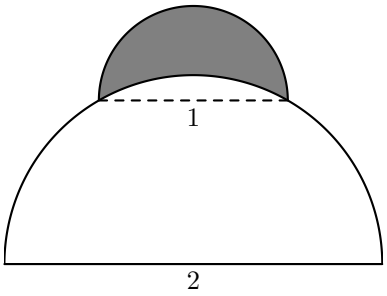
(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

14 Points K, L, M, and N lie in the plane of the square $ABCD$ so that AKB, BLC, CMD, and DNA are equilateral triangles. If $ABCD$ has an area of 16, nd the area of $KLMN$.



(A) 32 **(B)** $16+16\sqrt{3}$ **(C)** 48 **(D)** $32+16\sqrt{3}$ **(E)** 64

15 A semicircle of diameter 1 sits at the top of a semicircle of diameter 2, as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a lune. Determine the area of this lune.

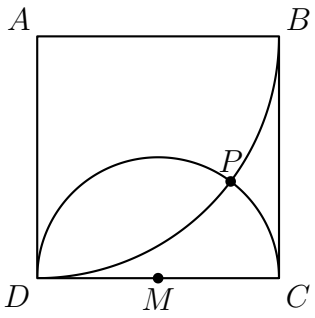


(A) $\frac{1}{6}\pi-\frac{\sqrt{3}}{4}$ **(B)** $\frac{\sqrt{3}}{4}-\frac{1}{12}\pi$ **(C)** $\frac{\sqrt{3}}{4}-\frac{1}{24}\pi$ **(D)** $\frac{\sqrt{3}}{4}+\frac{1}{24}\pi$ **(E)** $\frac{\sqrt{3}}{4}+\frac{1}{12}\pi$

16 A point P is chosen at random in the interior of equilateral triangle ABC. What is the probability that $\triangle ABP$ has a greater area than each of $\triangle ACP$ and $\triangle BCP$?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

17 Square $ABCD$ has sides of length 4, and M is the midpoint of $\overline{CD}$. A circle with radius 2 and center M intersects a circle with raidus 4 and center A at points P and D. What is the distance from P to $\overline{AD}$?



(A) 3 **(B)** $\frac{16}{5}$ **(C)** $\frac{13}{4}$ **(D)** $2\sqrt{3}$ **(E)** $\frac{7}{2}$

18 Let n be a 5-digit number, and let q and r be the quotient and remainder, respectively, when n is divided by 100. For how many values of n is $q + r$ divisible by 11?

(A) 8180 **(B)** 8181 **(C)** 8182 **(D)** 9000 **(E)** 9090

19 A parabola with equation $y = ax^2 + bx + c$ is reected about the x-axis. The parabola and its reection are translated horizontally ve units in opposite directions to become the graphs of $y = f(x)$ and $y = g(x)$, respectively. Which of the following describes the graph of $y = (f + g)(x)$?

(A) a parabola tangent to the x - axis **(B)** a parabola not tangent to the x - axis **(C)** a horizontal line **(D)** a non - horizontal line **(E)** the graph of a cubic function

20 How many 15-letter arrangements of 5 A's, 5 B's, and 5 C's have no A's in the first 5 letters, no B's in the next 5 letters, and no C's in the last 5 letters?

(A) $\sum_{k=0}^{5} \binom{5}{k}^3$ **(B)** $3^5 \cdot 2^5$ **(C)** 2^{15} **(D)** $\frac{15!}{(5!)^3}$ **(E)** 3^{15}

21 The graph of the polynomial

$P(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$

has five distinct x-intercepts, one of which is at $(0, 0)$. Which of the following coefficients cannot be zero?

(A) a **(B)** b **(C)** c **(D)** d **(E)** e

22 Objects A and B move simultaneously in the coordinate plane via a sequence of steps, each of length one. Object A starts at $(0,0)$ and each of its steps is either right or up, both equally likely. Object B starts at $(5,7)$ and each of its steps is either left or down, both equally likely. Which of the following is closest to the probability that the objects meet?

(A) 0.10 **(B)** 0.15 **(C)** 0.20 **(D)** 0.25 **(E)** 0.30

23 How many perfect squares are divisors of the product $1! \cdot 2! \cdot 3! \cdots 9!$?

(A) 504 **(B)** 672 **(C)** 864 **(D)** 936 **(E)** 1008

24 If $a \geq b > 1$, what is the largest possible value of $\log_a(a/b) + \log_b(b/a)$?

(A) -2 **(B)** 0 **(C)** 2 **(D)** 3 **(E)** 4

25 Let $f(x) = \sqrt{ax^2 + bx}$. For how many real values of a is there at least one positive value of b for which the domain of f and the range of f are the same set?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

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B

1 Which of the following is the same as

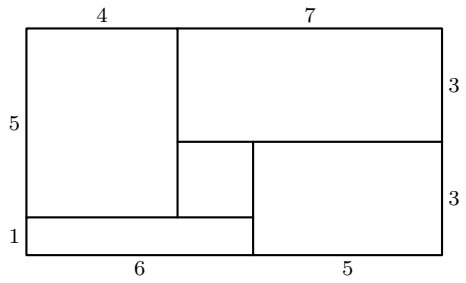
$$\frac{2-4+6-8+10-12+14}{3-6+9-12+15-18+21}?$$

(A) -1 **(B)** $-\frac{2}{3}$ **(C)** $\frac{2}{3}$ **(D)** 1 **(E)** $\frac{14}{3}$

2 Al gets the disease algebritis and must take one green pill and one pink pill each day for two weeks. A green pill costs \$1 more than a pink pill, and Als pills cost a total of \$546 for the two weeks. How much does one green pill cost?

(A) \$7 **(B)** \$14 **(C)** \$19 **(D)** \$20 **(E)** \$39

3 Rose fills each of the rectangular regions of her rectangular flower bed with a different type of flower. The lengths, in feet, of the rectangular regions in her flower bed are as shown in the gure. She plants one flower per square foot in each region. Asters cost \$1 each, begonias \$1.50 each, cannas \$2 each, dahlias \$2.50 each, and Easter lilies \$3 each. What is the least possible cost, in dollars, for her garden?

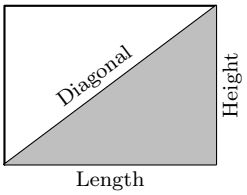


(A) 108 **(B)** 115 **(C)** 132 **(D)** 144 **(E)** 156

4 Moe uses a mower to cut his rectangular 90-foot by 150-foot lawn. The swath he cuts is 28 inches wide, but he overlaps each cut by 4 inches to make sure that no grass is missed. He walks at the rate of 5000 feet per hour while pushing the mower. Which of the following is closest to the number of hours it will take Moe to mow his lawn?

(A) 0.75 **(B)** 0.8 **(C)** 1.35 **(D)** 1.5 **(E)** 3

5 Many television screens are rectangles that are measured by the length of their diagonals. The ratio of the horizontal length to the height in a standard television screen is $4 : 3$. The horizontal length of a 27-inch television screen is closest, in inches, to which of the following?



(A) 20 **(B)** 20.5 **(C)** 21 **(D)** 21.5 **(E)** 22

6 The second and fourth terms of a geometric sequence are 2 and 6. Which of the following is a possible first term?

(A) $-\sqrt{3}$ **(B)** $-\frac{2\sqrt{3}}{3}$ **(C)** $-\frac{\sqrt{3}}{3}$ **(D)** $\sqrt{3}$ **(E)** 3

7 Penniless Petes piggy bank has no pennies in it, but it has 100 coins, all nickels, dimes, and quarters, whose total value is \$8.35. It does not necessarily contain coins of all three types. What is the difference between the largest and smallest number of dimes that could be in the bank?

(A) 0 **(B)** 13 **(C)** 37 **(D)** 64 **(E)** 83

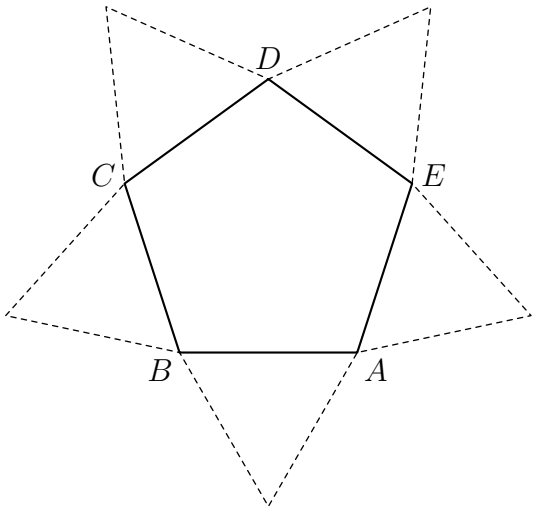
8 Let $\clubsuit(x)$ denote the sum of the digits of the positive integer x. For example, $\clubsuit(8) = 8$ and $\clubsuit(123) = 1 + 2 + 3 = 6$. For how many two-digit values of x is $\clubsuit(\clubsuit(x)) = 3$?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 9 **(E)** 10

9 Let f be a linear function for which $f(6) - f(2) = 12$. What is $f(12) - f(2)$?

(A) 12 **(B)** 18 **(C)** 24 **(D)** 30 **(E)** 36

10 Several gures can be made by attaching two equilateral triangles to the regular pentagon $ABCDE$ in two of the ve positions shown. How many non-congruent gures can be constructed in this way?



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

11 Cassandra sets her watch to the correct time at noon. At the actual time of 1: 00 PM, she notices that her watch reads 12: 57 and 36 seconds. Assuming that her watch loses time at a constant rate, what will be the actual time when her watch rst reads 10: 00 PM?

(A) 10: 22 PM and 24 seconds **(B)** 10: 24 PM **(C)** 10: 25 PM **(D)** 10: 27 PM **(E)** 10: 30 PM

12 What is the largest integer that is a divisor of

$$(n+1)(n+3)(n+5)(n+7)(n+9)$$

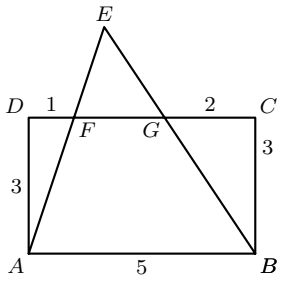
for all positive even integers n?

(A) 3 **(B)** 5 **(C)** 11 **(D)** 15 **(E)** 165

13 An ice cream cone consists of a sphere of vanilla ice cream and a right circular cone that has the same diameter as the sphere. If the ice cream melts, it will exactly ll the cone. Assume that the melted ice cream occupies 75% of the volume of the frozen ice cream. What is the ratio of the cones height to its radius?

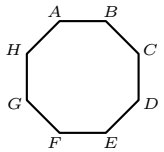
(A) 2 : 1 **(B)** 3 : 1 **(C)** 4 : 1 **(D)** 16 : 3 **(E)** 6 : 1

14 In rectangle $ABCD$, $AB = 5$ and $BC = 3$. Points F and G are on $\overline{CD}$ so that $DF = 1$ and $GC = 2$. Lines AF and BG intersect at E. Find the area of $\triangle AEB$.



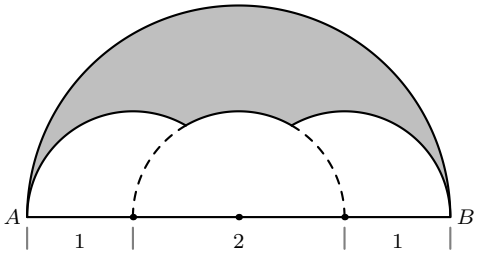
(A) 10 **(B)** $\frac{21}{2}$ **(C)** 12 **(D)** $\frac{25}{2}$ **(E)** 15

15 A regular octagon $ABCDEFGH$ has an area of one square unit. What is the area of the rectangle $ABEF$?



(A) $1-\frac{\sqrt{2}}{2}$ **(B)** $\frac{\sqrt{2}}{4}$ **(C)** $\sqrt{2}-1$ **(D)** $\frac{1}{2}$ **(E)** $\frac{1+\sqrt{2}}{4}$

16 Three semicircles of radius 1 are constructed on diameter AB of a semicircle of radius 2. The centers of the small semicircles divide $\overline{AB}$ into four line segments of equal length, as shown. What is the area of the shaded region that lies within the large semicircle but outside the smaller semicircles?



(A) $\pi-\sqrt{3}$ **(B)** $\pi-\sqrt{2}$ **(C)** $\frac{\pi+\sqrt{2}}{2}$ **(D)** $\frac{\pi+\sqrt{3}}{2}$ **(E)** $\frac{7}{6}\pi-\frac{\sqrt{3}}{2}$

17 If $\log(xy^3) = 1$ and $\log(x^2y) = 1$, what is $\log(xy)$?

(A) $-\frac{1}{2}$ **(B)** 0 **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{5}$ **(E)** 1

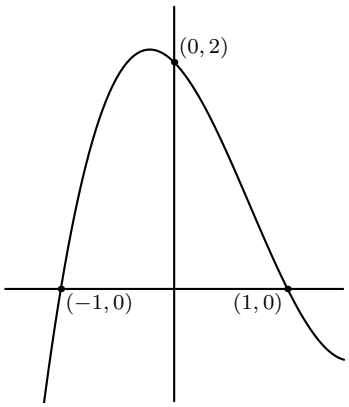
18 Let x and y be positive integers such that $7x^5 = 11y^{13}$. The minimum possible value of x has a prime factorization $a^c b^d$. What is $a + b + c + d$?

(A) 30 **(B)** 31 **(C)** 32 **(D)** 33 **(E)** 34

19 Let S be the set of permutations of the sequence $1, 2, 3, 4, 5$ for which the rst term is not 1. A permutation is chosen randomly from S. The probability that the second term is 2, in lowest terms, is a/b. What is $a + b$?

(A) 5 **(B)** 6 **(C)** 11 **(D)** 16 **(E)** 19

20 Part of the graph of $f(x) = x^3 + bx^2 + cx + d$ is shown. What is b?

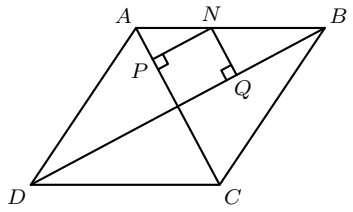


(A) -4 **(B)** -2 **(C)** 0 **(D)** 2 **(E)** 4

21 An object moves 8 cm in a straight line from A to B, turns at an angle α, measured in radians and chosen at random from the interval $(0, \pi)$, and moves 5 cm in a straight line to C. What is the probability that $AC < 7$?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

22 Let $ABCD$ be a rhombus with $AC = 16$ and $BD = 30$. Let N be a point on $\overline{AB}$, and let P and Q be the feet of the perpendiculars from N to $\overline{AC}$ and $\overline{BD}$, respectively. Which of the following is closest to the minimum possible value of PQ?



(A) 6.5 **(B)** 6.75 **(C)** 7 **(D)** 7.25 **(E)** 7.5

23 The number of x-intercepts on the graph of $y = \sin(1/x)$ in the interval $(0.0001, 0.001)$ is closest to

(A) 2900 **(B)** 3000 **(C)** 3100 **(D)** 3200 **(E)** 3300

24 Positive integers a, b, and c are chosen so that $a < b < c$, and the system of equations

$$2x + y = 2003 \text{ and } y = |x - a| + |x - b| + |x - c|$$

has exactly one solution. What is the minimum value of c?

(A) 668 **(B)** 669 **(C)** 1002 **(D)** 2003 **(E)** 2004

25 Three points are chosen randomly and independently on a circle. What is the probability that all three pairwise distances between the points are less than the radius of the circle?

(A) $\frac{1}{36}$ **(B)** $\frac{1}{24}$ **(C)** $\frac{1}{18}$ **(D)** $\frac{1}{12}$ **(E)** $\frac{1}{9}$