USA
AMC 12/AHSME
2007

A

1. One ticket to a show costs \$20 at full price. Susan buys 4 tickets using a coupon that gives her a 25% discount. Pam buys 5 tickets using a coupon that gives her a 30% discount. How many more dollars does Pam pay than Susan?

   (A) 2 **(B)** 5 **(C)** 10 **(D)** 15 **(E)** 20

2. An aquarium has a rectangular base that measures 100 cm by 40 cm and has a height of 50 cm. It is filled with water to a height of 40 cm. A brick with a rectangular base that measures 40 cm by 20 cm and a height of 10 cm is placed in the aquarium. By how many centimeters does the water rise?

   (A) 0.5 **(B)** 1 **(C)** 1.5 **(D)** 2 **(E)** 2.5

3. The larger of two consecutive odd integers is three times the smaller. What is their sum?

   (A) 4 **(B)** 8 **(C)** 12 **(D)** 16 **(E)** 20

4. Kate rode her bicycle for 30 minutes at a speed of 16 mph, then walked for 90 minutes at a speed of 4 mph. What was her overall average speed in miles per hour?

   (A) 7 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** 14

5. Last year Mr. John Q. Public received an inheritance. He paid 20% in federal taxes on the inheritance, and paid 10% of what he had left in state taxes. He paid a total of $\$10,500$ for both taxes. How many dollars was the inheritance?

   (A) $30,000$ **(B)** $32,500$ **(C)** $35,000$ **(D)** $37,500$ **(E)** $40,000$

6. Triangles ABC and ADC are isosceles with $AB = BC$ and $AD = DC$. Point D is inside $\triangle ABC$, $\angle ABC = 40^\circ$, and $\angle ADC = 140^\circ$. What is the degree measure of $\angle BAD$?

   (A) 20 **(B)** 30 **(C)** 40 **(D)** 50 **(E)** 60

7. Let a, b, c, d, and e be five consecutive terms in an arithmetic sequence, and suppose that $a + b + c + d + e = 30$. Which of the following can be found?

   (A) a **(B)** b **(C)** c **(D)** d **(E)** e

8. A star-polygon is drawn on a clock face by drawing a chord from each number to the firth number counted clockwise from that number. That is, chords are drawn from 12 to 5, from 5 to 10, from 10 to 3, and so on, ending back at 12. What is the degree measure of the angle at each vertex in the star-polygon?

   (A) 20 **(B)** 24 **(C)** 30 **(D)** 36 **(E)** 60

$\boxed{9}$ Yan is somewhere between his home and the stadium. To get to the stadium he can walk directly to the stadium, or else he can walk home and then ride his bicycle to the stadium. He rides 7 times as fast as he walks, and both choices require the same amount of time. What is the ratio of Yan's distance from his home to his distance from the stadium?

(A) $\frac{2}{3}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{4}{5}$ **(D)** $\frac{5}{6}$ **(E)** $\frac{6}{7}$

$\boxed{10}$ A triangle with side lengths in the ratio $3:4:5$ is inscribed in a circle of radius 3. What is the area of the triangle?

(A) 8.64 **(B)** 12 **(C)** 5π **(D)** 17.28 **(E)** 18

$\boxed{11}$ A finite sequence of three-digit integers has the property that the tens and units digits of each term are, respectively, the hundreds and tens digits of the next term, and the tens and units digits of the last term are, respectively, the hundreds and tens digits of the first term. For example, such a sequence might begin with the terms 247, 275, and 756 and end with the term 824. Let $\mathcal{S}$ be the sum of all the terms in the sequence. What is the largest prime factor that always divides $\mathcal{S}$?

(A) 3 **(B)** 7 **(C)** 13 **(D)** 37 **(E)** 43

$\boxed{12}$ Integers a, b, c, and d, not necessarily distinct, are chosen independantly and at random from 0 to 2007, inclusive. What is the probability that $ad-bc$ is even?

(A) $\frac{3}{8}$ **(B)** $\frac{7}{16}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{9}{16}$ **(E)** $\frac{5}{8}$

$\boxed{13}$ A piece of cheese is located at $(12,10)$ in a coordinate plane. A mouse is at $(4,-2)$ and is running up the line $y=-5x+18$. At the point (a,b) the mouse starts getting farther from the cheese rather than closer to it. What is $a+b$?

(A) 6 **(B)** 10 **(C)** 14 **(D)** 18 **(E)** 22

$\boxed{14}$ Let a, b, c, d, and e be distinct integers such that

$$(6-a)(6-b)(6-c)(6-d)(6-e)=45.$$

What is $a+b+c+d+e$?

(A) 5 **(B)** 17 **(C)** 25 **(D)** 27 **(E)** 30

$\boxed{15}$ The set $\{3,6,9,10\}$ is augmented by a fifth element n, not equal to any of the other four. The median of the resulting set is equal to its mean. What is the sum of all possible values of n?

(A) 7 **(B)** 9 **(C)** 19 **(D)** 24 **(E)** 26

16 How many three-digit numbers are composed of three distinct digits such that one digit is the average of the other two?

(A) 96 **(B)** 104 **(C)** 112 **(D)** 120 **(E)** 256

17 Suppose that $\sin a + \sin b = \sqrt{\frac{5}{3}}$ and $\cos a + \cos b = 1$. What is $\cos(a - b)$?

(A) $\sqrt{\frac{5}{3}} - 1$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** 1

18 The polynomial $f(x) = x^4 + ax^3 + bx^2 + cx + d$ has real coefficients, and $f(2i) = f(2+i) = 0$. What is $a + b + c + d$?

(A) 0 **(B)** 1 **(C)** 4 **(D)** 9 **(E)** 16

19 Triangles ABC and ADE have areas 2007 and 7002, respectively, with $B = (0, 0)$, $C = (223, 0)$, $D = (680, 380)$, and $E = (689, 389)$. What is the sum of all possible x-coordinates of A?

(A) 282 **(B)** 300 **(C)** 600 **(D)** 900 **(E)** 1200

20 Corners are sliced off a unit cube so that the six faces each become regular octagons. What is the total volume of the removed tetrahedra?

(A) $\frac{5\sqrt{2}-7}{3}$ **(B)** $\frac{10-7\sqrt{2}}{3}$ **(C)** $\frac{3-2\sqrt{2}}{3}$ **(D)** $\frac{8\sqrt{2}-11}{3}$ **(E)** $\frac{6-4\sqrt{2}}{3}$

21 The sum of the zeros, the product of the zeros, and the sum of the coefficients of the function $f(x) = ax^2 + bx + c$ are equal. Their common value must also be which of the following?

(A) the coefficient of x^2 **(B)** the coefficient of x **(C)** the y - intercept of the graph of $y = f(x)$ **(D)** one of the x - intercepts of the graph of $y = f(x)$ **(E)** the mean of the x - intercepts of the graph o $f(x)$

22 For each positive integer n, let $S(n)$ denote the sum of the digits of n. For how many values of n is $n + S(n) + S(S(n)) = 2007$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

23 Square $ABCD$ has area 36, and $\overline{AB}$ is parallel to the x-axis. Vertices A, B, and C are on the graphs of $y = \log_a x$, $y = 2\log_a x$, and $y = 3\log_a x$, respectively. What is a?

(A) $\sqrt[6]{3}$ **(B)** $\sqrt{3}$ **(C)** $\sqrt[3]{6}$ **(D)** $\sqrt{6}$ **(E)** 6

24 For each integer $n > 1$, let $F(n)$ be the number of solutions of the equation $\sin x = \sin nx$ on the interval $[0, \pi]$. What is $\sum_{n=2}^{2007} F(n)$?

(A) 2,014,524 **(B)** 2,015,028 **(C)** 2,015,033 **(D)** 2,016,532 **(E)** 2,017,033

$\boxed{25}$ Call a set of integers *spacy* if it contains no more than one out of any three consecutive integers. How many subsets of $\{1, 2, 3, \ldots, 12\}$, including the empty set, are spacy?

(A) 121 **(B)** 123 **(C)** 125 **(D)** 127 **(E)** 129

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B

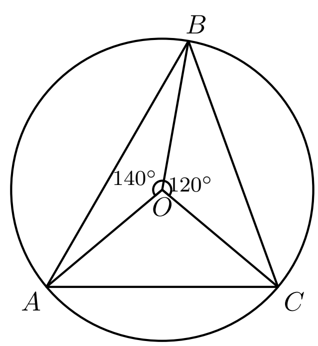
$\boxed{1}$ Isabella's house has 3 bedrooms. Each bedroom is 12 feet long, 10 feet wide, and 8 feet high. Isabella must paint the walls of all the bedrooms. Doorways and windows, which will not be painted, occupy 60 square feet in each bedroom. How many square feet of walls must be painted?

(A) 678 **(B)** 768 **(C)** 786 **(D)** 867 **(E)** 876

$\boxed{2}$ A college student drove his compact car 120 miles home for the weekend and averaged 30 miles per gallon. On the return trip the student drove his parents' SUV and averaged only 20 miles per gallon. What was the average gas mileage, in miles per gallon, for the round trip?

(A) 22 **(B)** 24 **(C)** 25 **(D)** 26 **(E)** 28

$\boxed{3}$ The point O is the center of the circle circumscribed about $\triangle ABC$, with $\angle BOC = 120^\circ$ and $\angle AOB = 140^\circ$, as shown. What is the degree measure of $\angle ABC$?



(A) 35 **(B)** 40 **(C)** 45 **(D)** 50 **(E)** 60

$\boxed{4}$ At Frank's Fruit Market, 3 bananas cost as much as 2 apples, and 6 apples cost as much as 4 oranges. How many oranges cost as much as 18 bananas?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 18

$\boxed{5}$ The 2007 AMC 12 contests will be scored by awarding 6 points for each correct response, 0 points for each incorrect response, and 1.5 points for each problem left unanswered. After looking over the 25 problems, Sarah has decided to attempt the first 22 and leave the last three unanswered. How many of the first 22 problems must she solve correctly in order to score at least 100 points?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

6 Triangle ABC has side lengths $AB = 5$, $BC = 6$, and $AC = 7$. Two bugs start simultaneously from A and crawl along the sides of the triangle in opposite directions at the same speed. They meet at point D. What is BD?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

7 All sides of the convex pentagon $ABCDE$ are of equal length, and $\angle A = \angle B = 90^\circ$. What is the degree measure of $\angle E$?

(A) 90 **(B)** 108 **(C)** 120 **(D)** 144 **(E)** 150

8 Tom's age is T years, which is also the sum of the ages of his three children. His age N years ago was twice the sum of their ages then. What is $\frac{T}{N}$?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

9 A function f has the property that $f(3x-1) = x^2 + x + 1$ for all real numbers x. What is $f(5)$?

(A) 7 **(B)** 13 **(C)** 31 **(D)** 111 **(E)** 211

10 Some boys and girls are having a car wash to raise money for a class trip to China. Initially 40% of the group are girls. Shortly thereafter two girls leave and two boys arrive, and then 30% of the group are girls. How many girls were initially in the group?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

11 The angles of quadrilateral $ABCD$ satisfy $\angle A = 2\angle B = 3\angle C = 4\angle D$. What is the degree measure of $\angle A$, rounded to the nearest whole number?

(A) 125 **(B)** 144 **(C)** 153 **(D)** 173 **(E)** 180

12 A teacher gave a test to a class in which 10% of the students are juniors and 90% are seniors. The average score on the test was 84. The juniors all received the same score, and the average score of the seniors was 83. What score did each of the juniors receive on the test?

(A) 85 **(B)** 88 **(C)** 93 **(D)** 94 **(E)** 98

13 A traffic light runs repeatedly through the following cycle: green for 30 seconds, then yellow for 3 seconds, and then red for 30 seconds. Leah picks a random three-second time interval to watch the light. What is the probability that the color changes while she is watching?

(A) $\frac{1}{63}$ **(B)** $\frac{1}{21}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{1}{7}$ **(E)** $\frac{1}{3}$

14 Point P is inside equilateral $\triangle ABC$. Points Q, R and S are the feet of the perpendiculars from P to $\overline{AB}, \overline{BC}$, and $\overline{CA}$, respectively. Given that $PQ = 1, PR = 2$, and $PS = 3$, what is AB?

(A) 4 **(B)** $3\sqrt{3}$ **(C)** 6 **(D)** $4\sqrt{3}$ **(E)** 9

15 The geometric series $a + ar + ar^2 + ...$ has a sum of 7, and the terms involving odd powers of r have a sum of 3. What is $a + r$?

(A) $\frac{4}{3}$ **(B)** $\frac{12}{7}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{7}{3}$ **(E)** $\frac{5}{2}$

16 Each face of a regular tetrahedron is painted either red, white or blue. Two colorings are considered indistinguishable if two congruent tetrahedra with those colorings can be rotated so that their appearances are identical. How many distinguishable colorings are possible?

(A) 15 **(B)** 18 **(C)** 27 **(D)** 54 **(E)** 81

17 If a is a nonzero integer and b is a positive number such that $ab^2 = \log_{10} b$, what is the median of the set $\{0, 1, a, b, 1/b\}$?

(A) 0 **(B)** 1 **(C)** a **(D)** b **(E)** $\frac{1}{b}$

18 Let a, b, and c be digits with $a \neq 0$. The three-digit integer abc lies one third of the way from the square of a positive integer to the square of the next larger integer. The integer acb lies two thirds of the way between the same two squares. What is $a + b + c$?

(A) 10 **(B)** 13 **(C)** 16 **(D)** 18 **(E)** 21

19 Rhombus $ABCD$, with a side length 6, is rolled to form a cylinder of volume 6 by taping $\overline{AB}$ to $\overline{DC}$. What is $\sin(\angle ABC)$?

(A) $\frac{\pi}{9}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{\pi}{6}$ **(D)** $\frac{\pi}{4}$ **(E)** $\frac{\sqrt{3}}{2}$

20 The parallelogram bounded by the lines $y = ax + c, y = ax + d, y = bx + c$ and $y = bx + d$ has area 18. The parallelogram bounded by the lines $y = ax + c, y = ax - d, y = bx + c$, and $y = bx - d$ has area 72. Given that a, b, c, and d are positive integers, what is the smallest possible value of $a + b + c + d$?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

21 The first 2007 positive integers are each written in base 3. How many of these base-3 representations are palindromes? (A palindrome is a number that reads the same forward and backward.)

(A) 100 **(B)** 101 **(C)** 102 **(D)** 103 **(E)** 104

22 Two particles move along the edges of equilateral triangle $\triangle ABC$ in the direction

$$A \to B \to C \to A$$

starting simultaneously and moving at the same speed. One starts at A, and the other starts at the midpoint of $\overline{BC}$. The midpoint of the line segment joining the two particles traces out a path that encloses a region R. What is the ratio of the area of R to the area of $\triangle ABC$?

(A) $\frac{1}{16}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{9}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{4}$

$\boxed{23}$ How many non-congruent right triangles with positive integer leg lengths have areas that are numerically equal to 3 times their perimeters?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 10 **(E)** 12

$\boxed{24}$ How many pairs of positive integers (a, b) are there such that $\gcd(a, b) = 1$ and

$$\frac{a}{b} + \frac{14b}{9a}$$

is an integer?

(A) 4 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** infinitely many

$\boxed{25}$ Points A, B, C, D, and E are located in 3-dimensional space with $AB = BC = CD = DE = EA = 2$ and $\angle ABC = \angle CDE = \angle DEA = 90^\circ$. The plane of $\triangle ABC$ is parallel to $\overline{DE}$. What is the area of $\triangle BDE$?

(A) $\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** 2 **(D)** $\sqrt{5}$ **(E)** $\sqrt{6}$