USA
AMC 12/AHSME
2010

A

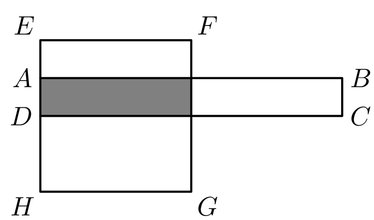
1 What is $(20 - (2010 - 201)) + (2010 - (201 - 20))$?

(A) -4020 **(B)** 0 **(C)** 40 **(D)** 401 **(E)** 4020

2 A ferry boat shuttles tourists to an island every hour starting at 10 AM until its last trip, which starts at 3 PM. One day the boat captain notes that on the 10 AM trip there were 100 tourists on the ferry boat, and that on each successive trip, the number of tourists was 1 fewer than on the previous trip. How many tourists did the ferry take to the island that day?

(A) 585 **(B)** 594 **(C)** 672 **(D)** 679 **(E)** 694

3 Rectangle $ABCD$, pictured below, shares 50



(A) 4 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 10

4 If $x < 0$, then which of the following must be positive?

(A) $\frac{x}{|x|}$ **(B)** $-x^2$ **(C)** -2^x **(D)** $-x^{-1}$ **(E)** $\sqrt[3]{x}$

5 Halfway through a 100-shot archery tournament, Chelsea leads by 50 points. For each shot a bullseye scores 10 points, with other possible scores being $8, 4, 2, 0$ points. Chelsea always scores at least 4 points on each shot. If Chelsea's next n shots are bulleyes she will be guaranteed victory. What is the minimum value for n?

(A) 38 **(B)** 40 **(C)** 42 **(D)** 44 **(E)** 46

6 A *palindrome*, such as 83438, is a number that remains the same when its digits are reversed. The numbers x and $x + 32$ are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

7 Logan is constructing a scaled model of his town. The city's water tower stands 40 meters high, and the top portion is a sphere that holes 100,000 liters of water. Logan's miniature water tower holds 0.1 liters. How tall, in meters, should Logan make his tower?

(A) 0.04 **(B)** $\frac{0.4}{\pi}$ **(C)** 0.4 **(D)** $\frac{4}{\pi}$ **(E)** 4

8 Triangle ABC has $AB = 2 \cdot AC$. Let D and E be on $\overline{AB}$ and $\overline{BC}$, respectively, such that $\angle BAE = \angle ACD$. Let F be the intersection of segments AE and CD, and suppose that $\triangle CFE$ is equilateral. What is $\angle ACB$?

(A) 60° **(B)** 75° **(C)** 90° **(D)** 105° **(E)** 120°

9 A solid cube has side length 3 inches. A 2-inch by 2-inch square hole is cut into the center of each face. The edges of each cut are parallel to the edges of the cube, and each hole goes all the way through the cube. What is the volume, in cubic inches, of the remaining solid?

(A) 7 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 15

10 The first four terms of an arithmetic sequence are $p, 9, 3p-q$, and $3p+q$. What is the 2010^{th} term of the sequence?

(A) 8041 **(B)** 8043 **(C)** 8045 **(D)** 8047 **(E)** 8049

11 The solution of the equation $7^{x+7} = 8^x$ can be expressed in the form $x = \log_b 7^7$. What is b?

(A) $\frac{7}{15}$ **(B)** $\frac{7}{8}$ **(C)** $\frac{8}{7}$ **(D)** $\frac{15}{8}$ **(E)** $\frac{15}{7}$

12 In a magical swamp there are two species of talking amphibians: toads, whose statements are always true, and frogs, whose statements are always false. Four amphibians, Brian, Chris, LeRoy, and Mike live together in the swamp, and they make the following statements:

Brian: "Mike and I are different species." Chris: "LeRoy is a frog." LeRoy: "Chris is a frog." Mike: "Of the four of us, at least two are toads."

How many of these amphibians are frogs?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

13 For how many integer values of k do the graphs of $x^2 + y^2 = k^2$ and $xy = k$ not intersect?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 8

14 Nondegenerate $\triangle ABC$ has integer side lengths, BD is an angle bisector, $AD = 3$, and $DC = 8$. What is the smallest possible value of the perimeter?

(A) 30 **(B)** 33 **(C)** 35 **(D)** 36 **(E)** 37

15 A coin is altered so that the probability that it lands on heads is less than $\frac{1}{2}$ and when the coin is flipped four times, the probability of an equal number of heads and tails is $\frac{1}{6}$. What is the probability that the coin lands on heads?

(A) $\frac{\sqrt{15}-3}{6}$ **(B)** $\frac{6-\sqrt{6\sqrt{6}+2}}{12}$ **(C)** $\frac{\sqrt{2}-1}{2}$ **(D)** $\frac{3-\sqrt{3}}{6}$ **(E)** $\frac{\sqrt{3}-1}{2}$

16 Bernardo randomly picks 3 distinct numbers from the set $\{1,2,3,4,5,6,7,8,9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1,2,3,4,5,6,7,8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

(A) $\frac{47}{72}$ **(B)** $\frac{37}{56}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{49}{72}$ **(E)** $\frac{39}{56}$

17 Equiangular hexagon $ABCDEF$ has side lengths $AB = CD = EF = 1$ and $BC = DE = FA = r$. The area of $\triangle ACE$ is 70

(A) $\frac{4\sqrt{3}}{3}$ **(B)** $\frac{10}{3}$ **(C)** 4 **(D)** $\frac{17}{4}$ **(E)** 6

18 A 16-step path is to go from $(-4,-4)$ to $(4,4)$ with each step increasing either the x-coordinate or the y-coordinate by 1. How many such paths stay outside or on the boundary of the square $-2 \le x \le 2$, $-2 \le y \le 2$ at each step?

(A) 92 **(B)** 144 **(C)** 1568 **(D)** 1698 **(E)** 12,800

19 Each of 2010 boxes in a line contains a single red marble, and for $1 \le k \le 2010$, the box in the kth position also contains k white marbles. Isabella begins at the first box and successively draws a single marble at random from each box, in order. She stops when she first draws a red marble. Let $P(n)$ be the probability that Isabella stops after drawing exactly n marbles. What is the smallest value of n for which $P(n) < \frac{1}{2010}$?

(A) 45 **(B)** 63 **(C)** 64 **(D)** 201 **(E)** 1005

20 Arithmetic sequences (a_n) and (b_n) have integer terms with $a_1 = b_1 = 1 < a_2 \le b_2$ and $a_n b_n = 2010$ for some n. What is the largest possible value of n?

(A) 2 **(B)** 3 **(C)** 8 **(D)** 288 **(E)** 2009

21 The graph of $y = x^6 - 10x^5 + 29x^4 - 4x^3 + ax^2$ lies above the line $y = bx + c$ except at three values of x, where the graph and the line intersect. What is the largest of those values?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

22 What is the minimum value of $f(x) = |x-1| + |2x-1| + |3x-1| + \cdots + |119x-1|$?

(A) 49 **(B)** 50 **(C)** 51 **(D)** 52 **(E)** 53

23 The number obtained from the last two nonzero digits of $90!$ is equal to n. What is n?

(A) 12 **(B)** 32 **(C)** 48 **(D)** 52 **(E)** 68

24 Let $f(x) = \log_{10}(\sin(\pi x) \cdot \sin(2\pi x) \cdot \sin(3\pi x) \cdots \sin(8\pi x))$. The intersection of the domain of $f(x)$ with the interval $[0, 1]$ is a union of n disjoint open intervals. What is n?

(A) 2 **(B)** 12 **(C)** 18 **(D)** 22 **(E)** 36

25 Two quadrilaterals are considered the same if one can be obtained from the other by a rotation and a translation. How many different convex cyclic quadrilaterals are there with integer sides and perimeter equal to 32?

(A) 560 **(B)** 564 **(C)** 568 **(D)** 1498 **(E)** 2255

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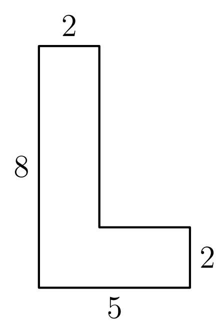
B

1 Makayla attended two meetings during her 9-hour work day. The first meeting took 45 minutes and the second meeting took twice as long. What percent of her work day was spent attending meetings?

(A) 15 **(B)** 20 **(C)** 25 **(D)** 30 **(E)** 35

2 A big L is formed as shown. What is its area?



(A) 22 **(B)** 24 **(C)** 26 **(D)** 28 **(E)** 30

3 A ticket to a school play costs x dollars, where x is a whole number. A group of 9th graders buys tickets costing a total of \$48, and a group of 10th graders buys tickets costing a total of \$64. How many values of x are possible?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

4 A month with 31 days has the same number of Mondays and Wednesdays. How many of the seven days of the week could be the first day of this month?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

5 Lucky Larry's teacher asked him to substitute numbers for a, b, c, d, and e in the expression $a-(b-(c-(d+e)))$ and evaluate the result. Larry ignored the parentheses but added and subtracted correctly and obtained the correct result by coincidence. The numbers Larry substituted for a, b, c, and d were 1, 2, 3, and 4, respectively. What number did Larry substitute for e?

(A) -5 **(B)** -3 **(C)** 0 **(D)** 3 **(E)** 5

6 At the beginning of the school year, 50% of all students in Mr. Well's math class answered "Yes" to the question "Do you love math", and 50% answered "No." At the end of the school year, 70% answered "Yes" and 30% answered "No." Altogether, $x\%$ of the students gave a different answer at the beginning and end of the school year. What is the difference between the maximum and the minimum possible values of x?

(A) 0 **(B)** 20 **(C)** 40 **(D)** 60 **(E)** 80

7 Shelby drives her scooter at a speed of 30 miles per hour if it is not raining, and 20 miles per hour if it is raining. Today she drove in the sun in the morning and in the rain in the evening, for a total of 16 miles in 40 minutes. How many minutes did she drive in the rain?

(A) 18 **(B)** 21 **(C)** 24 **(D)** 27 **(E)** 30

8 Every high school in the city of Euclid sent a team of 3 students to a math contest. Each participant in the contest received a different score. Andrea's score was the median among all students, and hers was the highest score on her team. Andrea's teammates Beth and Carla placed 37th and 64th, respectively. How many schools are in the city?

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

9 Let n be the smallest positive integer such that n is divisible by 20, n^2 is a perfect cube, and n^3 is a perfect square. What is the number of digits of n?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

10 The average of the numbers $1, 2, 3, ..., 98, 99$, and x is $100x$. What is x?

(A) $\frac{49}{101}$ **(B)** $\frac{50}{101}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{51}{101}$ **(E)** $\frac{50}{99}$

11 A palindrome between 1000 and $10,000$ is chosen at random. What is the probability that it is divisible by 7?

(A) $\dfrac{1}{10}$ **(B)** $\dfrac{1}{9}$ **(C)** $\dfrac{1}{7}$ **(D)** $\dfrac{1}{6}$ **(E)** $\dfrac{1}{5}$

12 For what value of x does

$$\log_{\sqrt{2}}\sqrt{x} + \log_2 x + \log_4(x^2) + \log_8(x^3) + \log_{16}(x^4) = 40?$$

(A) 8 **(B)** 16 **(C)** 32 **(D)** 256 **(E)** 1024

13 In $\triangle ABC$, $\cos(2A - B) + \sin(A + B) = 2$ and $AB = 4$. What is BC?

(A) $\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** 2 **(D)** $2\sqrt{2}$ **(E)** $2\sqrt{3}$

14 Let a, b, c, d, and e be positive integers with $a + b + c + d + e = 2010$, and let M be the largest of the sums $a + b$, $b + c$, $c + d$, and $d + e$. What is the smallest possible value of M?

(A) 670 **(B)** 671 **(C)** 802 **(D)** 803 **(E)** 804

15 For how many ordered triples (x, y, z) of nonnegative integers less than 20 are there exactly two distinct elements in the set $\{i^x, (1+i)^y, z\}$, where $i = \sqrt{-1}$?

(A) 149 **(B)** 205 **(C)** 215 **(D)** 225 **(E)** 235

16 Positive integers a, b, and c are randomly and independently selected with replacement from the set $\{1, 2, 3, \ldots, 2010\}$. What is the probability that $abc + ab + a$ is divisible by 3?

(A) $\dfrac{1}{3}$ **(B)** $\dfrac{29}{81}$ **(C)** $\dfrac{31}{81}$ **(D)** $\dfrac{11}{27}$ **(E)** $\dfrac{13}{27}$

17 The entries in a 3×3 array include all the digits from 1 through 9, arranged so that the entries in every row and column are in increasing order. How many such arrays are there?

(A) 18 **(B)** 24 **(C)** 36 **(D)** 42 **(E)** 60

18 A frog makes 3 jumps, each exactly 1 meter long. The directions of the jumps are chosen independently and at random. What is the probability the the frog's final position is no more than 1 meter from its starting position?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

19 A high school basketball game between the Raiders and Wildcats was tied at the end of the first quarter. The number of points scored by the Raiders in each of the four quarters formed an increasing geometric sequence, and the number of points scored by the Wildcats in each of the four quarters formed an increasing arithmetic sequence. At the end of the fourth quarter, the Raiders had won by one point. Neither team scored more than 100 points. What was the total number of points scored by the two teams in the first half?

(A) 30 **(B)** 31 **(C)** 32 **(D)** 33 **(E)** 34

20 A geometric sequence (a_n) has $a_1 = \sin x, a_2 = \cos x$, and $a_3 = \tan x$ for some real number x. For what value of n does $a_n = 1 + \cos x$?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

21 Let $a > 0$, and let $P(x)$ be a polynomial with integer coefficients such that

$$P(1) = P(3) = P(5) = P(7) = a, \text{ and}$$

$$P(2) = P(4) = P(6) = P(8) = -a.$$

What is the smallest possible value of a?

(A) 105 **(B)** 315 **(C)** 945 **(D)** 7! **(E)** 8!

22 Let $ABCD$ be a cyclic quadrilateral. The side lengths of $ABCD$ are distinct integers less than 15 such that $BC \cdot CD = AB \cdot DA$. What is the largest possible value of BD?

(A) $\sqrt{\frac{325}{2}}$ **(B)** $\sqrt{185}$ **(C)** $\sqrt{\frac{389}{2}}$ **(D)** $\sqrt{\frac{425}{2}}$ **(E)** $\sqrt{\frac{533}{2}}$

23 Monic quadratic polynomials $P(x)$ and $Q(x)$ have the property that $P(Q(x))$ has zeroes at $x = -23, -21, -17, \text{and} -15$, and $Q(P(x))$ has zeroes at $x = -59, -57, -51, \text{and} -49$. What is the sum of the minimum values of $P(x)$ and $Q(x)$?

(A) -100 **(B)** -82 **(C)** -73 **(D)** -64 **(E)** 0

24 The set of real numbers x for which

$$\frac{1}{x-2009} + \frac{1}{x-2010} + \frac{1}{x-2011} \geq 1$$

is the union of intervals of the form $a < x \leq b$. What is the sum of the lengths of these intervals?

(A) $\frac{1003}{335}$ **(B)** $\frac{1004}{335}$ **(C)** 3 **(D)** $\frac{403}{134}$ **(E)** $\frac{202}{67}$

25 For every integer $n \geq 2$, let $\text{pow}(n)$ be the largest power of the largest prime that divides n. For example $\text{pow}(144) = \text{pow}(2^4 \cdot 3^2) = 3^2$. What is the largest integer m such that 2010^m divides

$$\prod_{n=2}^{5300} \text{pow}(n)?$$

(A) 74 **(B)** 75 **(C)** 76 **(D)** 77 **(E)** 78