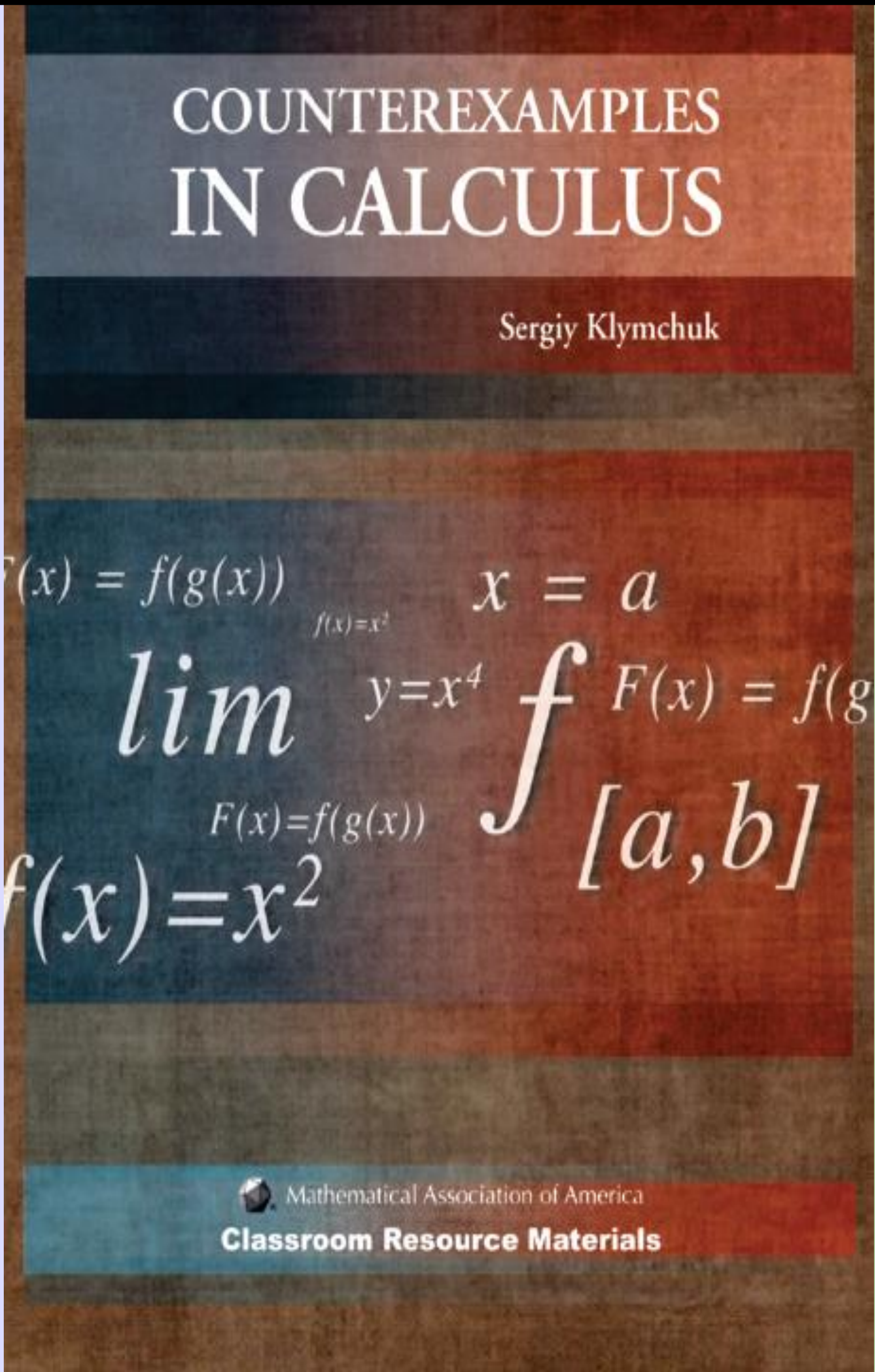
CONTRAEXEMPLE ÎN ANALIZA MATEMATICA

2016



Nicolae Coman

Traducere după lucrarea «Counterexamples in Calculus» de Serghii Klîmciuk

1/18/2016

Cuprins

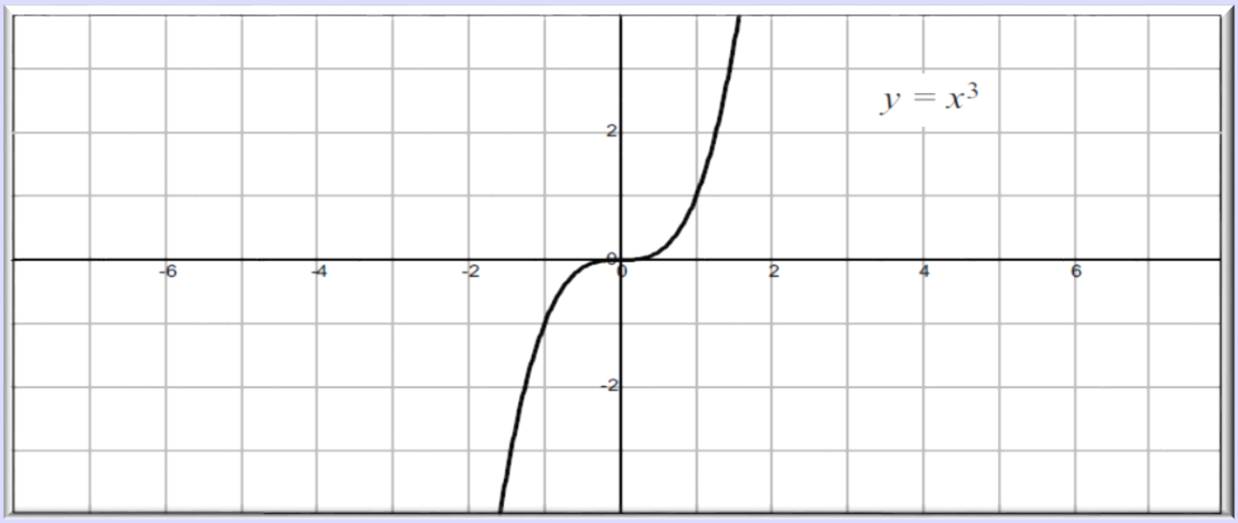
0. Introducere

1. Funcţii

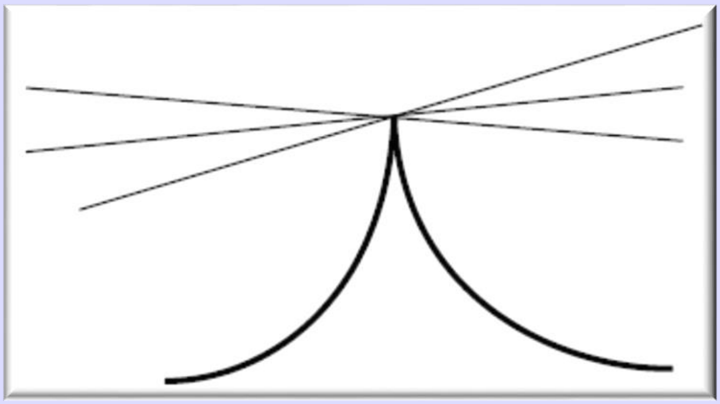
1. 1. Tangenta la o curbă într-un punct al acesteia este dreapta care atinge curba în acel punct dar nu o intersectează acolo.

Contraexemple.

a) Axa Ox este tangentă la curba $y = x^3$, dar o intersectează în origine.



b) Cele trei drepte intersectează curba din imaginea de mai jos, dar nu sunt tangente la curbă.



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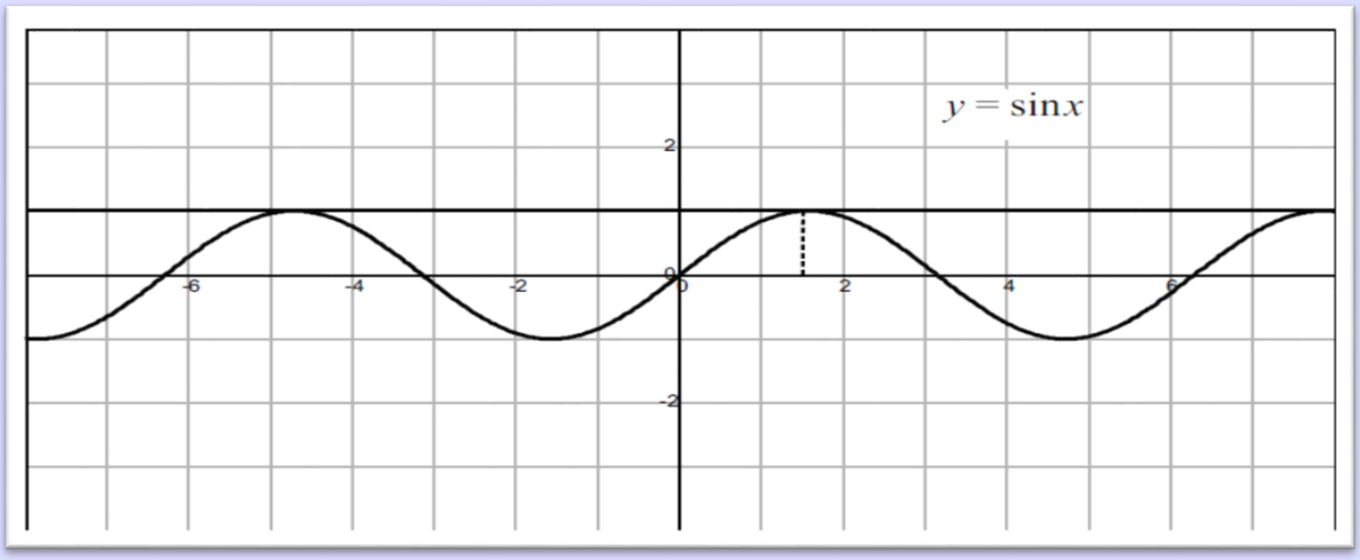
1. 2. Tangenta la o curbă într-un punct ale acestei nu poate intersecta curba într-o infinitate de puncte.

Contraexemplu. Tangenta la curba $y = \sin x$ intersectează curba în $x = \frac{\pi}{2}$ şi într-o infinitate de alte puncte.



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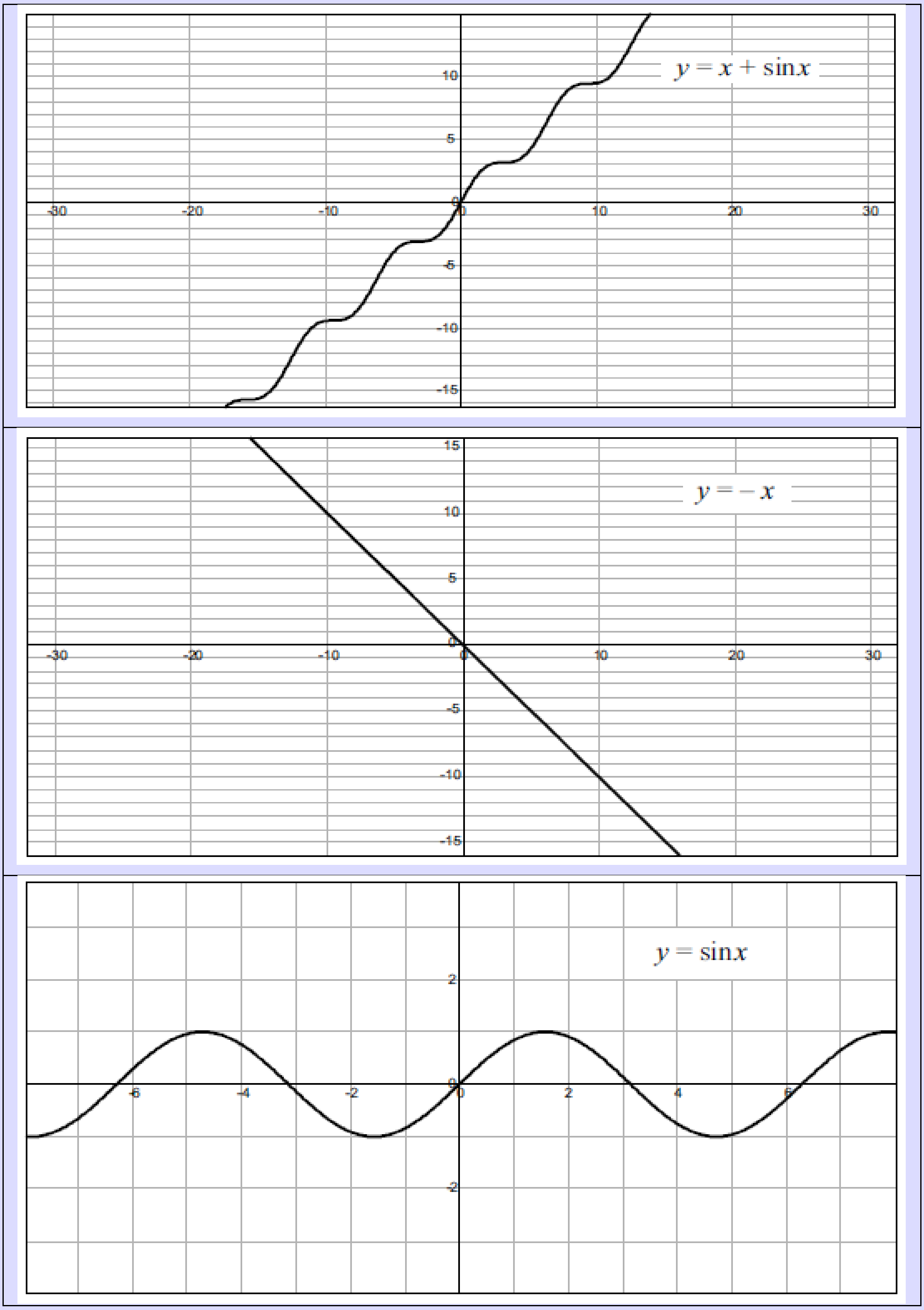
1. 3. O funcţie de gradul al doilea de x este acea funcţie în care cea mai mare putere a lui x este 2.
Contraexemplu. În ambele funcţii de mai jos, cea mai mare putere a lui x este 2, dar niciuna nu este de gradul al doilea:

$$y = x^2 + \sqrt{x}\,; \quad y = x^2 + x - \frac{1}{x}.$$

1. 4. Dacă funcţiile $f(x)$ şi $g(x)$ sunt continue şi monotone pe $\mathbb{R}$, atunci suma acestora, $f(x) + g(x)$, este de asemenea monotonă pe $\mathbb{R}$.
Contraexemplu. Considerăm funcţiile date de:

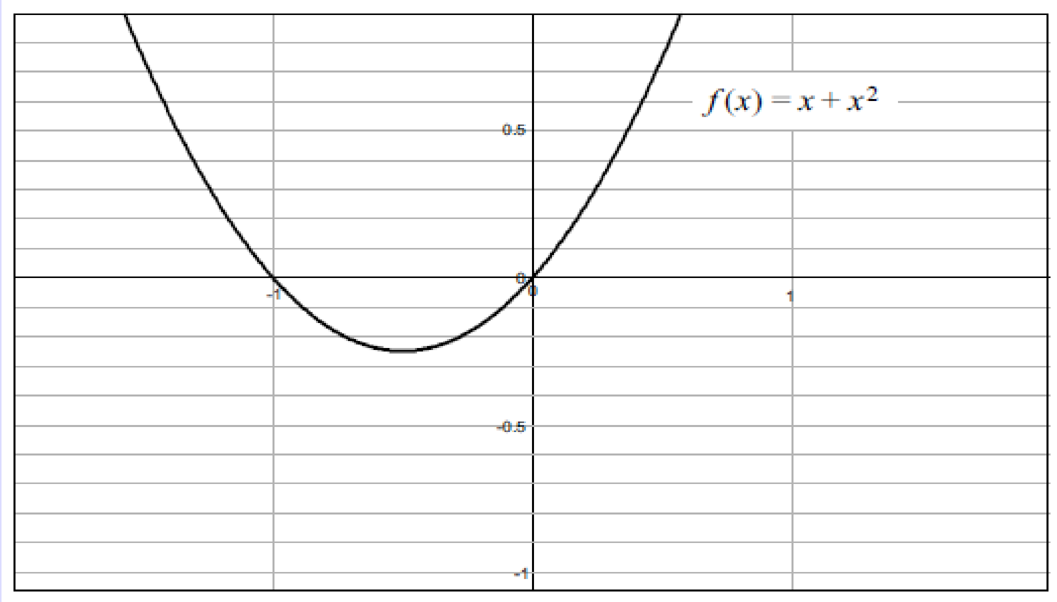
$$f(x) = x + \sin x\,, \; g(x) = -x\,.$$

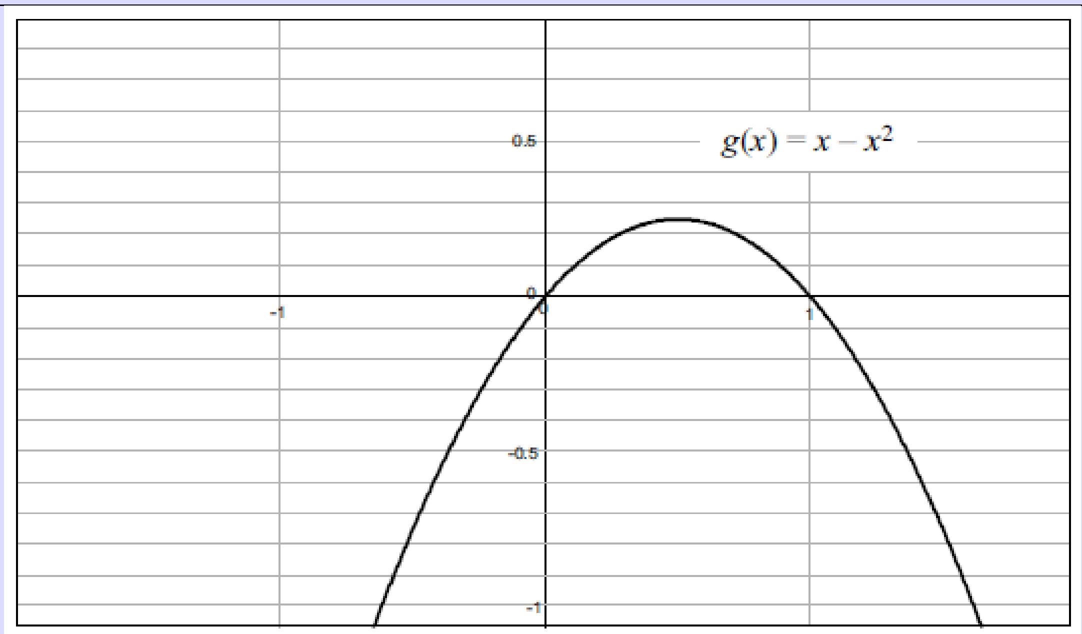
Ambele sunt monotone pe $\mathbb{R}$, dar suma lor nu este monotonă pe $\mathbb{R}$.

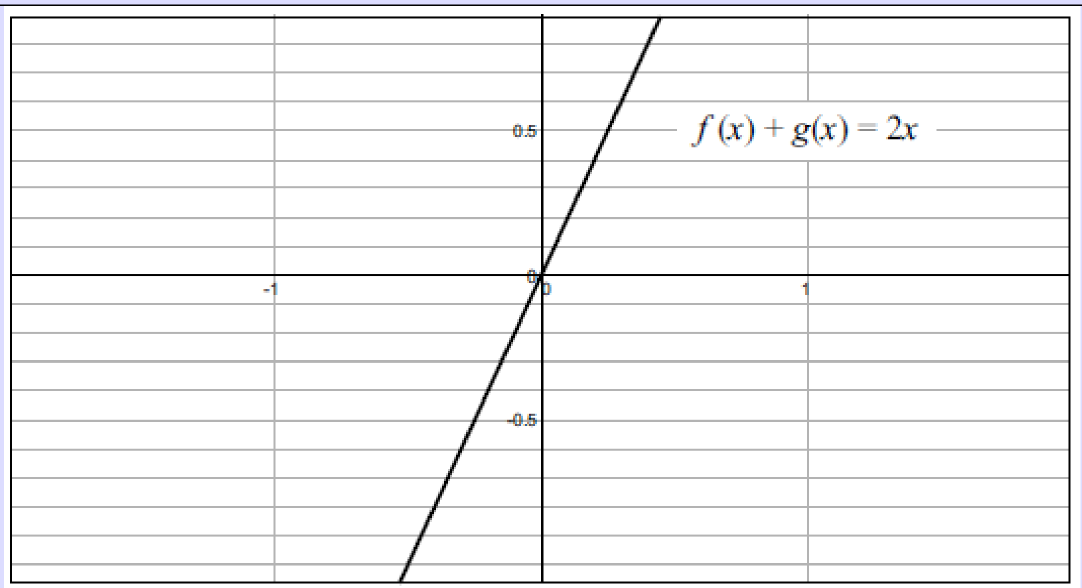


1. 5. Dacă funcţiile $f(x)$ şi $g(x)$ nu sunt monotone pe $\mathbb{R}$, atunci nici suma lor, $f(x)+g(x)$, nu este monotonă pe $\mathbb{R}$.

<u>**Contraexemplu.**</u> Funcţiile $f(x)=x+x^2$ şi $g(x)=x-x^2$ nu sunt monotone pe $\mathbb{R}$, dar suma lor, $f(x)+g(x)=2x$, este monotonă pe $\mathbb{R}$.

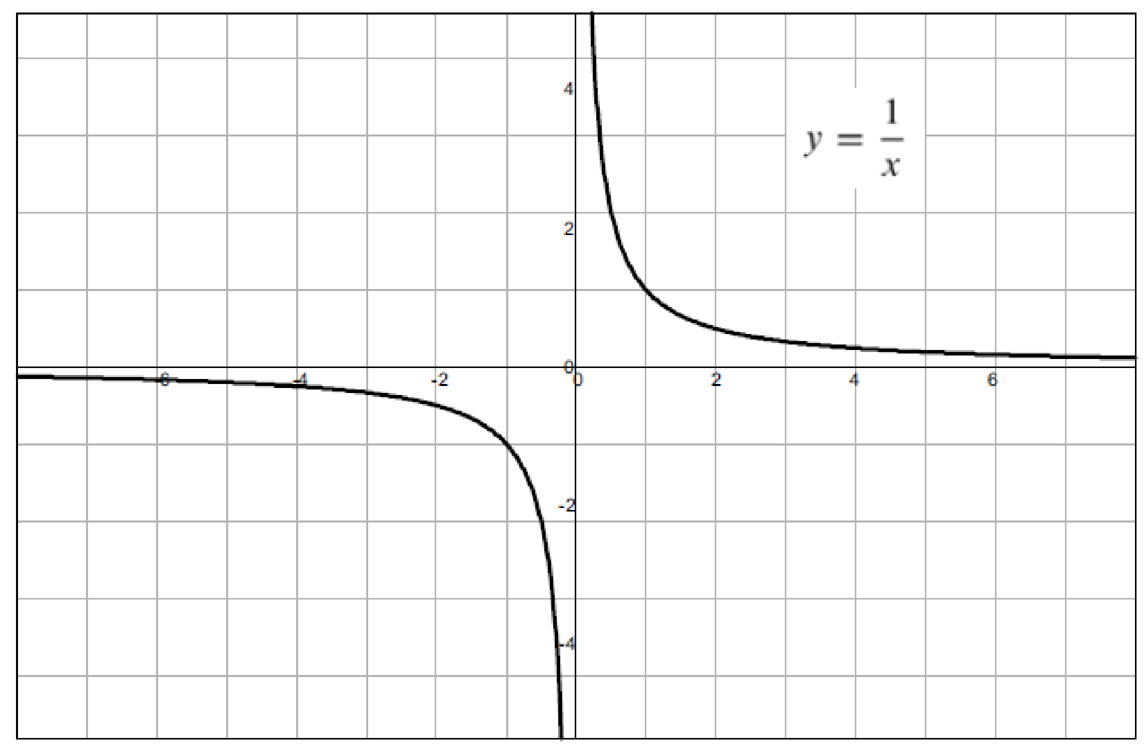






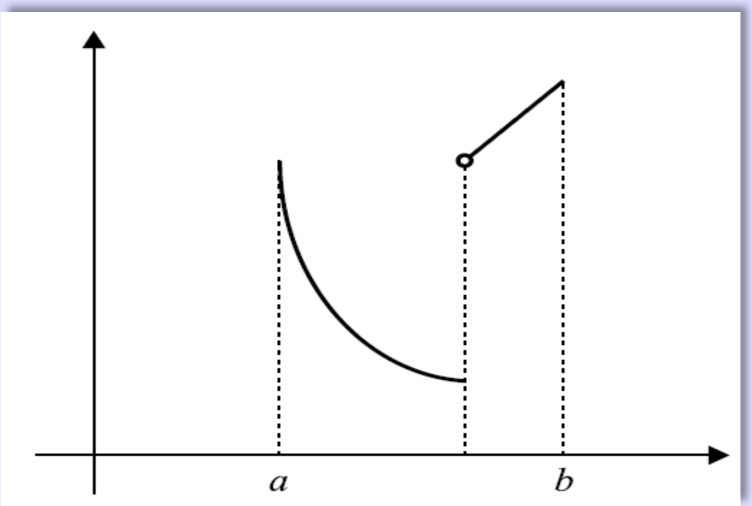
1.6. Dacă o funcţie $f(x)$ este continuă şi descrescătoare pentru orice $x > 0$ şi $f(1) > 0$, atunci funcţia are exact o rădăcină.

Contraexemplu. Funcţia $y = \frac{1}{x}$ este continuă şi descrescătoare pentru orice $x > 0$ şi $f(1) = 1 > 0$, dar nu are rădăcini reale.



1.7. Dacă o funcţie $f(x)$ admite inversa $f^{-1}(y)$ pe (a, b), atunci funcţia $f(x)$ este fie crescătoare, fie descrescătoare pe (a, b).

Contraexemplu. Funcţia de mai jos este o funcţie bijectivă pe (a, b) şi care deci admite inversă pe acest interval, dar nu este o funcţie monotonă.



1.8. O funcţie $f(x)$ este mărginită pe $\mathbb{R}$ dacă pentru orice $x \in \mathbb{R}$ există un $M > 0$ astfel încât $|f(x)| \leq M$.

Contraexemplu. Pentru funcţia $y = x^2$, pentru orice $x \in \mathbb{R}$, există un $M > 0$ $(M = x^2 + \varepsilon,$ unde $\varepsilon \geq 0$)astfel încât $|f(x)| \leq M$.

Comentariu. De aici reiese că ordinea cuvintelor într-o afirmaţie este foarte importantă. Definiţia corectă a funcţiei mărginite pe $\mathbb{R}$ diferă doar prin ordinea cuvintelor:

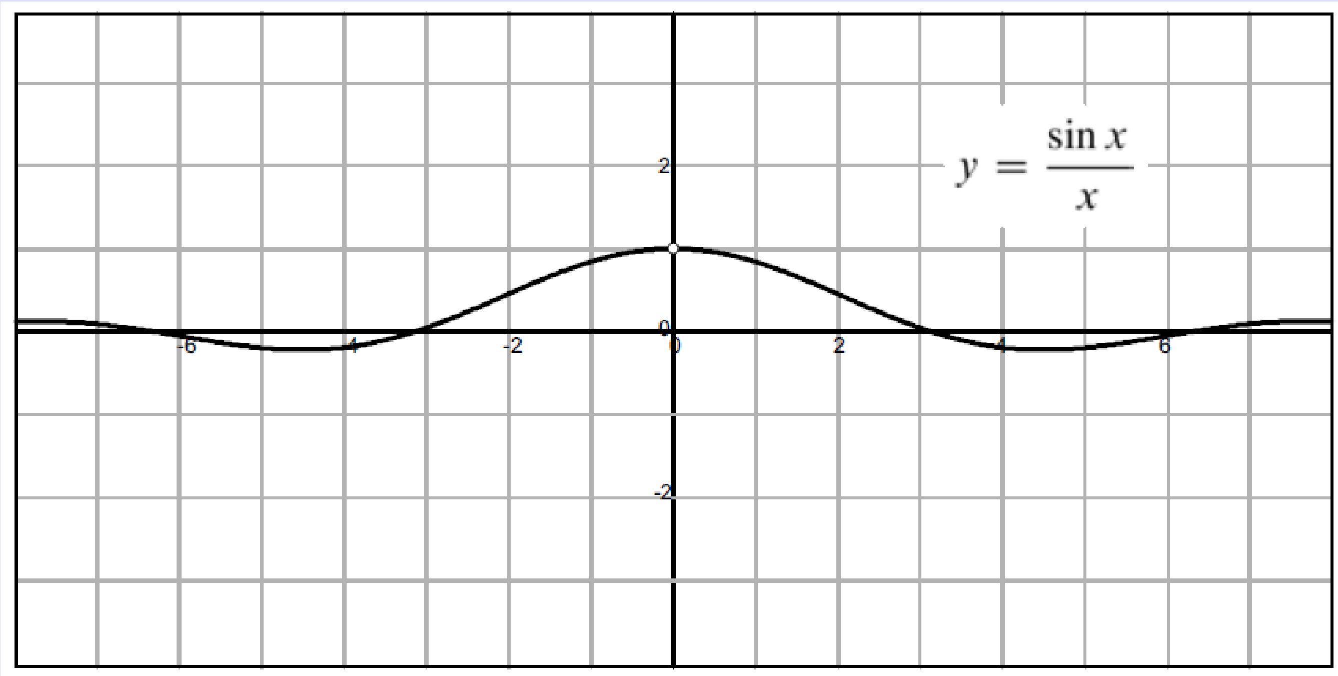
O funcţie $f(x)$ este mărginită pe $\mathbb{R}$ dacă există un $M > 0$ astfel încât pentru orice $x \in \mathbb{R}$, $|f(x)| \leq M$.

1.9. Dacă $g(a) = 0$, atunci funcţia dată de $F(x) = \frac{f(x)}{g(x)}$ admite o asimptotă verticală în punctul $x = a$.

Contraexemplu. Funcţia

$$y = \frac{\sin x}{x}$$

nu admite asimptotă verticală în punctul $x = 0$.

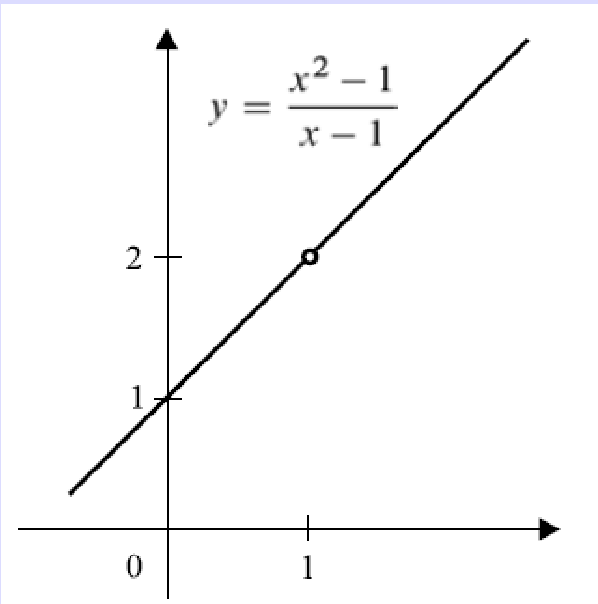


1.10. Dacă $g(a) = 0$, atunci funcţia *raţională* $R(x) = \frac{f(x)}{g(x)}$ (unde $f(x)$ şi $g(x)$ sunt funcţii polinomiale) admite o asimptotă verticală în punctul $x = a$.

Contraexemplu. Funcţia raţională

$$y = \frac{x^2 - 1}{x - 1}$$

nu admite asimptotă verticală în $x = 1$.



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2. Limite

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3. Continuitate

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4. Calcul diferenţial

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5. Calcul integral

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6. Anexă

Culoarea paginii 220, 220, 255.