b) $f'(1) = \lim\limits_{\Delta x \to 0} \frac{\sqrt[5]{1+\Delta x} - 1}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{1}{\sqrt[5]{(\Delta x)^4}} = \infty$; c) $f'_-\left(\frac{2k+1}{2}\pi\right) =$

$= \lim\limits_{\Delta x \to -0} \frac{\left|\cos\left(\frac{2k+1}{2}\pi + \Delta x\right)\right|}{\Delta x} = \lim\limits_{\Delta x \to -0} \frac{|\sin \Delta x|}{\Delta x} = -1$; $f'_+\left(\frac{2k+1}{2}\pi\right) =$

$= \lim\limits_{\Delta x \to +0} \frac{|\sin \Delta x|}{\Delta x} = 1$. **368.** $5x^4 - 12x^2 + 2$. **369.** $-\frac{1}{3} + 2x - 2x^3$. **370.** $2ax + b$.

371. $-\frac{15x^2}{a}$. **372.** $mat^{m-1} + b(m+n)t^{m+n-1}$. **373.** $\frac{6ax^5}{\sqrt{a^2+b^2}}$. **374.** $-\frac{\pi}{x^2}$.

375. $2x^{-\frac{1}{3}} - 5x^{\frac{3}{2}} - 3x^{-4}$. **376.** $\frac{8}{3}x^{\frac{5}{3}}$. Hint. $y = x^2 x^{\frac{2}{3}} = x^{\frac{8}{3}}$. **377.** $\frac{4b}{3x^2 \sqrt[3]{x}} -$

$- \frac{2a}{3x\sqrt[3]{x^2}}$. **378.** $\frac{bc-ad}{(c+dx)^2}$. **379.** $\frac{-2x^2-6x+25}{(x^2-5x+5)^2}$. **380.** $\frac{1-4x}{x^2(2x-1)^2}$.

381. $\frac{1}{\sqrt{z}(1-\sqrt{z})^2}$. **382.** $5\cos x - 3\sin x$. **383.** $\frac{4}{\sin^2 2x}$. **384.** $\frac{-2}{(\sin x - \cos x)^2}$.

385. $t^2 \sin t$. **386.** $y' = 0$. **387.** $\cot x - \frac{x}{\sin^2 x}$. **388.** $\arcsin x + \frac{x}{\sqrt{1-x^2}}$.

389. $x \arctan x$. **390.** $x^6 e^x (x+7)$. **391.** xe^x. **392.** $e^x \frac{x-2}{x^3}$. **393.** $\frac{5x^4 - x^5}{e^x}$.

394. $e^x(\cos x - \sin x)$. **395.** $x^2 e^x$. **396.** $e^x\left(\arcsin x + \frac{1}{\sqrt{1-x^2}}\right)$. **397.** $\frac{x(2\ln x - 1)}{\ln^2 x}$.

398. $3x^2 \ln x$. **399.** $\frac{2}{x} + \frac{\ln x}{x^2} - \frac{2}{x^2}$. **400.** $\frac{2\ln x}{x \ln 10} - \frac{1}{x}$. **401.** $\sinh x + x\cosh x$.

402. $\frac{2x\cosh x - x^2 \sinh x}{\cosh^2 x}$. **403.** $-\tanh^2 x$. **404.** $\frac{-3(x\ln x + \sinh x \cosh x)}{x \ln^2 x \cdot \sinh^2 x}$.

405. $\frac{-2x^2}{1-x^4}$. **406.** $\frac{1}{\sqrt{1-x^2}} \operatorname{arc\,sinh} x + \frac{1}{\sqrt{1+x^2}} \arcsin x$.

407. $\frac{x - \sqrt{x^2-1}\operatorname{arc\,cosh} x}{x^2\sqrt{x^2-1}}$. **408.** $\frac{1+2x\operatorname{arc\,tanh} x}{(1-x^2)^2}$. **410.** $\frac{3a}{c}\left(\frac{ax+b}{c}\right)^2$.

411. $12ab + 18b^2y$. **412.** $16x(3+2x^2)^3$. **413.** $\frac{x^2-1}{(2x-1)^8}$. **414.** $\frac{-x}{\sqrt{1-x^2}}$.

415. $\frac{bx^2}{\sqrt[3]{(a+bx^3)^2}}$. **416.** $-\sqrt{\sqrt[3]{\frac{a^2}{x^2}} - 1}$. **418.** $\frac{1-\tan^2 x + \tan^4 x}{\cos^2 x}$.

419. $\frac{-1}{2\sin^2 x \sqrt{\cot x}}$. **420.** $2 - 15\cos^2 x \sin x$. **421.** $\frac{-16\cos 2t}{\sin^3 2t}$. Hint. $x = \sin^{-2} t +$

$+ \cos^{-2} t$. **422.** $\frac{\sin x}{(1-3\cos x)^3}$. **423.** $\frac{\sin^3 x}{\cos^4 x}$. **424.** $\frac{3\cos x + 2\sin x}{2\sqrt{15\sin x - 10\cos x}}$.

425. $\frac{2\cos x}{3\sqrt[3]{\sin x}} + \frac{3\sin x}{\cos^4 x}$. **426.** $\frac{1}{2\sqrt{1-x^2}\sqrt{1+\arcsin x}}$.

427. $\frac{1}{2(1+x^2)\sqrt{\arctan x}} - \frac{3(\arcsin x)^2}{\sqrt{1-x^2}}$. **428.** $\frac{-1}{(1+x^2)(\arctan x)^2}$.

429. $\frac{e^x+xe^x+1}{2\sqrt{xe^x+x}}$. 430. $\frac{2e^x-2^x\ln 2}{3\sqrt[3]{(2e^x-2^x+1)^2}}+\frac{5\ln^4 x}{x}$. 432. $(2x-5)\times$ $\times\cos(x^5-5x+1)-\frac{a}{x^2\cos^2\frac{a}{x}}$. 433. $-\alpha\sin(\alpha x+\beta)$. 434. $\sin(2t+\varphi)$.
435. $-2\frac{\cos x}{\sin^3 x}$. 436. $\frac{-1}{\sin^2\frac{x}{a}}$. 437. $x\cos 2x^2\sin 3x^2$. 438. **Solution.** $\frac{1}{\sqrt{1-(2x)^2}}(2x)'=\frac{2}{\sqrt{1-4x^2}}$. 439. $\frac{-2}{x\sqrt{x^4-1}}$. 440. $\frac{-1}{2\sqrt{x-x^2}}$. 441. $\frac{-1}{1+x^2}$.
442. $\frac{-1}{1+x^2}$. 443. $-10xe^{-x^2}$. 444. $-2x5^{-x^2}\ln 5$. 445. $2x10^{2x}(1+x\ln 10)$.
446. $\sin 2^t+2^t t\cos 2^t\ln 2$. 447. $\frac{-e^x}{\sqrt{1-e^{2x}}}$. 448. $\frac{2}{2x+7}$. 449. $\cot x\log e$.
450. $\frac{-2x}{1-x^2}$. 451. $\frac{2\ln x}{x}-\frac{1}{x\ln x}$. 452. $\frac{(e^x+5\cos x)\sqrt{1-x^2}-4}{(e^x+5\sin x-4\arcsin x)\sqrt{1-x^2}}$.
453. $\frac{1}{(1+\ln^2 x)x}+\frac{1}{(1+x^2)\arctan x}$. 454. $\frac{1}{2x\sqrt{\ln x+1}}+\frac{1}{2(\sqrt{x}+x)}$.
455. **Solution.** $y'=(\sin^3 5x)'\cos^2\frac{x}{3}+\sin^3 5x\left(\cos^2\frac{x}{3}\right)'=3\sin^2 5x\cos 5x\,5\cos^2\frac{x}{3}+$ $+\sin^3 5x\,2\cos\frac{x}{3}\left(-\sin\frac{x}{3}\right)\frac{1}{3}=15\sin^2 5x\cos 5x\cos^2\frac{x}{3}-\frac{2}{3}\sin^3 5x\cos\frac{x}{3}\sin\frac{x}{3}$.
456. $\frac{4x+3}{(x-2)^3}$ 457. $\frac{x^2+4x-6}{(x-3)^5}$. 458. $\frac{x^7}{(1-x^2)^5}$. 459. $\frac{x-1}{x^2\sqrt{2x^2-2x+1}}$.
460. $\frac{1}{\sqrt{(a^2+x^2)^3}}$. 461. $\frac{x^2}{\sqrt{(1+x^2)^5}}$. 462. $\frac{(1+\sqrt{x})^3}{\sqrt[3]{x}}$. 463. $x^5\sqrt[3]{(1+x^3)^2}$.
464. $\frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}$. 465. $4x^3(a-2x^3)(a-5x^3)$.
466. $\frac{2abmnx^{n-1}(a+bx^n)^{m-1}}{(a-bx^n)^{m+1}}$. 467. $\frac{x^3-1}{(x+2)^6}$. 468. $\frac{a-3x}{2\sqrt{a-x}}$.
469. $\frac{3x^2+2(a+b+c)x+ab+bc+ac}{2\sqrt{(x+a)(x+b)(x+c)}}$. 470. $\frac{1+2\sqrt{y}}{6\sqrt{y}\sqrt[3]{(y+\sqrt{y})^2}}$.
471. $2(7t+4)\sqrt[3]{3t+2}$. 472. $\frac{y-a}{\sqrt{(2ay-y^2)^3}}$. 473. $\frac{1}{\sqrt{e^x+1}}$. 474. $\sin^3 x\cos^2 x$.
475. $\frac{1}{\sin^4 x\cos^4 x}$. 476. $10\tan 5x\sec^2 5x$. 477. $x\cos x^2$. 478. $3t^2\sin 2t^3$.
479. $3\cos x\cos 2x$. 480. $\tan^4 x$. 481. $\frac{\cos 2x}{\sin^4 x}$. 482. $\frac{(\alpha-\beta)\sin 2x}{2\sqrt{\alpha\sin^2 x+\beta\cos^2 x}}$. 483. 0.
484. $\frac{1}{2}\frac{\arcsin x(2\arccos x-\arcsin x)}{\sqrt{1-x^2}}$. 485. $\frac{2}{x\sqrt{2x^2-1}}$. 486. $\frac{1}{1+x^2}$.
487. $\frac{x\arccos x-\sqrt{1-x^2}}{(1-x^2)^{3/2}}$. 488. $\frac{1}{\sqrt{a-bx^2}}$. 489. $\sqrt{\frac{a-x}{a+x}}$. 490. $2\sqrt{a^2-x^2}$.
491. $\frac{-x}{\sqrt{2x-x^2}}$. 492. $\arcsin\sqrt{x}$. 493. $\frac{5}{\sqrt{1-25x^2}\arcsin 5x}$.

494. $\frac{1}{x\sqrt{1-\ln^2 x}}$. 495. $\frac{\sin\alpha}{1-2x\cos\alpha+x^2}$. 496. $\frac{1}{5+4\sin x}$.
497. $4x\sqrt{\frac{x}{b-x}}$. 498. $\frac{\sin^2 x}{1+\cos^2 x}$. 499. $\frac{a}{2}\sqrt{e^{ax}}$. 500. $\sin 2xe^{\sin^2 x}$.
501. $2m^2p\,(2ma^{mx}+b)^{p-1}a^{mx}\ln a$. 502. $e^{\alpha t}(\alpha\cos\beta t-\beta\sin\beta t)$. 503. $e^{\alpha x}\sin\beta x$.
504. $e^{-x}\cos 3x$. 505. $x^{n-1}a^{-x^2}(n-2x^2\ln a)$. 506. $-\frac{1}{2}y\tan x\,(1+\sqrt{\cos x}\ln a)$.
507. $\frac{3\cot\frac{1}{x}\ln 3}{\left(x\sin\frac{1}{x}\right)^2}$. 508. $\frac{2ax+b}{ax^2+bx+c}$. 509. $\frac{1}{\sqrt{a^2+x^2}}$. 510. $\frac{\sqrt{x}}{1+\sqrt{x}}$.
511. $\frac{1}{\sqrt{2ax+x^2}}$. 512. $\frac{-2}{x\ln^3 x}$. 513. $-\frac{1}{x^2}\tan\frac{x-1}{x}$. 514. $\frac{2x+11}{x^2-x-2}$. Hint. $y=5\ln(x-2)-3\ln(x+1)$. 515. $\frac{3x^2-16x+19}{(x-1)(x-2)(x-3)}$. 516. $\frac{1}{\sin^3 x\cos x}$.
517. $\sqrt{x^2-a^2}$. 518. $\frac{-6x^2}{(3-2x^3)\ln(3-2x^3)}$. 519. $\frac{15a\ln^2(ax+b)}{ax+b}$.
520. $\frac{2}{\sqrt{x^2+a^2}}$. 521. $\frac{mx+n}{x^2-a^2}$. 522. $\sqrt{2}\sin\ln x$. 523. $\frac{1}{\sin^3 x}$.
524. $\frac{\sqrt{1+x^2}}{x}$. 525. $\frac{x+1}{x^3-1}$. 526. $\frac{3}{\sqrt{1-9x^2}}[2^{\arcsin 3x}\ln 2+2(1-\arccos 3x)]$.
527. $\left(3^{\frac{\sin ax}{\cos bx}}\ln 3+\frac{\sin^2 ax}{\cos^2 bx}\right)\frac{a\cos ax\cos bx+b\sin ax\sin bx}{\cos^2 bx}$. 528. $\frac{1}{1+2\sin x}$.
529. $\frac{1}{x(1+\ln^2 x)}$. 530. $\frac{1}{\sqrt{1-x^2}\arcsin x}+\frac{\ln x}{x}+\frac{1}{x\sqrt{1-\ln^2 x}}$.
531. $-\frac{1}{x(1+\ln^2 x)}$. 532. $\frac{x^2}{x^4+x^2-2}$. 533. $\frac{2}{\cos x\sqrt{\sin x}}$. 534. $\frac{x^2-3x}{x^4-1}$.
535. $\frac{1}{1+x^3}$. 536. $\frac{\arcsin x}{(1-x^2)^{3/2}}$. 537. $6\sinh^2 2x\cdot\cosh 2x$. 538. $e^{\alpha x}(\alpha\cosh\beta x+$ $+\beta\sinh\beta x)$. 539. $6\tanh^2 2x\,(1-\tanh^2 2x)$ 540. $2\coth 2x$. 541. $\frac{2x}{\sqrt{a^4+x^4}}$.
542. $\frac{1}{x\sqrt{\ln^2 x-1}}$ 543. $\frac{1}{\cos 2x}$. 544. $\frac{-1}{\sin x}$. 545. $\frac{2}{1-x^2}$. 546. $x\,\mathrm{arc\,tanh}\,x$
547. $x\,\mathrm{arc\,sinh}\,x$. 548. a) $y'=1$ when $x>0$; $y'=-1$ when $x<0$; $y'(0)$ does not exist; b) $y'=|2x|$. 549. $y'=\frac{1}{x}$. 550. $f'(x)=\begin{cases}-1 \text{ when } x\leqslant 0,\\ -e^{-x} \text{ when } x>0.\end{cases}$
552. $\frac{1}{2}+\frac{\sqrt{3}}{3}$. 553. 6π. 554. a) $f'_-(0)=-1$, $f'_+(0)=1$; b) $f'_-(0)=\frac{2}{a}$, $f'_+(0)=\frac{-2}{a}$; c) $f'_-(0)=1$, $f'_+(0)=0$; d) $f'_-(0)=f'_+(0)=0$, e) $f'_-(0)$ and $f'_+(0)$ do not exist. 555. $1-x$. 556. $2+\frac{x-3}{4}$. 557. -1. 558. 0 561. **Solution.** We have $y'=e^{-x}(1-x)$. Since $e^{-x}=\frac{y}{x}$, it follows that $y'=\frac{y}{x}(1-x)$ or $xy'=y(1-x)$ 566. $(1+2x)(1+3x)+2(1+x)(1+3x)+3(x+1)(1+2x)$.
567. $-\frac{(x+2)(5x^2+19x+20)}{(x+1)^4(x+3)^5}$. 568. $\frac{x^2-4x+2}{2\sqrt{x(x-1)(x-2)^3}}$

569. $\frac{3x^2+5}{3(x^2+1)}\sqrt[3]{\frac{x^2}{x^2+1}}$. **570.** $\frac{(x-2)^9(x^2-7x+1)}{(x-1)(x-2)(x-3)\sqrt{(x-1)^5(x-3)^4}}$.
571. $-\frac{5x^2+x-24}{3(x-1)^{1/2}(x+2)^{5/3}(x+3)^{5/2}}$. **572.** $x^x(1+\ln x)$. **573.** $x^{x^2+1}(1+2\ln x)$.
574. $\sqrt[x]{x}\,\frac{1-\ln x}{x^2}$. **575.** $x^{\sqrt{x}-\frac{1}{2}}\left(1+\frac{1}{2}\ln x\right)$. **576.** $x^{x^x}x^x\left(\frac{1}{x}+\ln x+\ln^2 x\right)$.
577. $x^{\sin x}\left(\frac{\sin x}{x}+\cos x\ln x\right)$. **578.** $(\cos x)^{\sin x}(\cos x\ln\cos x-\sin x\tan x)$.
579. $\left(1+\frac{1}{x}\right)^x\left[\ln\left(1+\frac{1}{x}\right)+\frac{1}{1+x}\right]$. **580.** $(\arctan x)^x\times$ $\times\left[\ln\arctan x+\frac{x}{(1+x^2)\arctan x}\right]$. **581.** a) $x'_y=\frac{1}{3(1+x^2)}$;
b) $x'_y=\frac{2}{2-\cos x}$; c) $x'_y=\frac{10}{1+5e^{\frac{x}{2}}}$. **582.** $\frac{3}{2}t^2$. **583.** $\frac{t-1}{t+1}$. **584.** $\frac{-2t}{1-t^2}$.
585. $\frac{t(2-t^3)}{1-2t^3}$. **586.** $\frac{2}{3\sqrt[6]{t}}$. **587.** $\frac{t+1}{t(t^2+1)}$. **588.** $\tan t$. **589.** $-\frac{b}{a}$.
590. $-\frac{b}{a}\tan t$. **591.** $-\tan 3t$. **592.** $y'_x=\begin{cases}-1 \text{ when } t<0,\\ \ \ 1 \text{ when } t>0.\end{cases}$ **593.** $-2e^{3t}$.
594. $\tan t$. **596.** 1. **597.** ∞. **599.** No. **600.** Yes, since the equality is an identity. **601.** $\frac{2}{5}$. **602.** $-\frac{b^2x}{a^2y}$. **603.** $-\frac{x^2}{y^2}$. **604.** $-\frac{x(3x+2y)}{x^2+2y}$. **605.** $-\sqrt{\frac{y}{x}}$.
606. $-\sqrt[3]{\frac{y}{x}}$. **607.** $\frac{2y^2}{3(x^2-y^2)+2xy}=\frac{1-y^3}{1+3xy^2+4y^3}$. **608.** $\frac{10}{10-3\cos y}$.
609. -1. **610.** $\frac{y\cos^2 y}{1-x\cos^2 y}$. **611.** $\frac{y}{x}\,\frac{1-x^2-y^2}{1+x^2+y^2}$. **612.** $(x+y)^2$. **613.** $y'=$ $=\frac{1}{e^y-1}=\frac{1}{x+y-1}$. **614.** $\frac{y}{x}+e^{\frac{y}{x}}$. **615.** $\frac{y}{x-y}$. **616.** $\frac{x+y}{x-y}$.
617. $\frac{cy+x\sqrt{x^2+y^2}}{cx-y\sqrt{x^2+y^2}}$. **618.** $\frac{x\ln y-y}{y\ln x-x}\,\frac{y}{x}$. **620.** a) 0; b) $\frac{1}{2}$; c) 0. **622.** 45°; $\arctan 2\approx 63°26'$. **623.** 45°. **624.** $\arctan\frac{2}{e}\approx 36°21'$. **625.** (0, 20); (1, 15); (−2, −12). **626.** (1, −3). **627.** $y=x^2-x+1$. **628.** $k=\frac{-1}{11}$. **629.** $\left(\frac{1}{8},\ -\frac{1}{16}\right)$.
631. $y-5=0$; $x+2=0$. **632.** $x-1=0$; $y=0$. **633.** a) $y=2x$; $y=-\frac{1}{2}x$;
b) $x-2y-1=0$; $2x+y-2=0$; c) $6x+2y-\pi=0$; $2x-6y+3\pi=0$;
d) $y=x-1$; $y=1-x$; e) $2x+y-3=0$; $x-2y+1=0$ for the point (1, 1); $2x-y+3=0$; $x+2y-1=0$ for the point (−1, 1). **634.** $7x-10y+6=0$, $10x+7y-34=0$. **635.** $y=0$; $(\pi+4)x+(\pi-4)y-\frac{\pi^2\sqrt{2}}{4}=0$. **636.** $5x+6y-$ $-13=0$, $6x-5y+21=0$. **637.** $x+y-2=0$. **638.** At the point (1, 0): $y=2x-2$; $y=\frac{1-x}{2}$; at the point (2, 0): $y=-x+2$; $y=x-2$; at the point (3, 0): $y=2x-6$; $y=\frac{3-x}{2}$. **639.** $14x-13y+12=0$; $13x+14y-41=0$.

640. **Hint**. The equation of the tangent is $\frac{x}{2x_0}+\frac{y}{2y_0}=1$. Hence, the tangent crosses the x-axis at the point $A(2x_0, 0)$ and the y-axis at $B(0, 2y_0)$. Finding the midpoint of AB, we get the point (x_0, y_0). **643**. 40° 36′. **644**. The parabolas are tangent at the point (0, 0) and intersect at an angle $\arctan\frac{1}{7}\approx 8°\,8'$ at the point (1, 1). **647**. $S_t=S_n=2$; $t=n=2\sqrt{2}$. **648**. $\frac{1}{\ln 2}$. **652**. $T=2a\sin\frac{t}{2}\tan\frac{t}{2}$; $N=2a\sin\frac{t}{2}$; $S_t=2a\sin^2\frac{t}{2}\tan\frac{t}{2}$; $S_n=a\sin t$. **653**. $\arctan\frac{1}{k}$. **654**. $\frac{\pi}{2}+2\varphi$. **655**. $S_t=4\pi^2 a$; $S_n=a$; $t=2\pi a\sqrt{1+4\pi^2}$; $n=a\sqrt{1+4\pi^2}$; $\tan\mu=2\pi$. **656**. $S_t=a$; $S_n=\frac{a}{\varphi_0^2}$; $t=\sqrt{a^2+\varrho_0^2}$; $n=\frac{\varrho_0}{a}\sqrt{a^2+\varrho_0^2}$; $\tan\mu=-\varphi_0$. **657**. 3 cm/sec; 0; —9 cm/sec **658**. 15 cm/sec. **659**. $-\frac{3}{2}$ m/sec. **660**. The equation of the trajectory is $y=x\tan\alpha-\frac{g}{2v_0^2\cos^2\alpha}x^2$. The range is $\frac{v_0^2\sin 2\alpha}{g}$. The velocity, $\sqrt{v_0^2-2v_0gt\sin\alpha+g^2t^2}$; the slope of the velocity vector is $\frac{v_0\sin\alpha-gt}{v_0\cos\alpha}$. **Hint**. To determine the trajectory, eliminate the parameter t from the given system. The range is the abscissa of the point A (Fig. 17). The projections of velocity on the axes are $\frac{dx}{dt}$ and $\frac{dy}{dt}$ The magnitude of the velocity is $\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$; the velocity vector is directed along the tangent to the trajectory **661**. Diminishes with the velocity 0.4 **62**. $\left(\frac{9}{8}, \frac{9}{2}\right)$. **663**. The diagonal increases at a rate of ~ 3.8 cm/sec, the area, at a rate of 40 cm²/sec **664**. The surface area increases at a rate of 0.2π m²/sec, the volume, at a rate of 0.05π m³/sec. **665**. $\frac{\pi}{3}$ cm/sec **666**. The mass of the rod is 360 g, the density at M is $5x$ g/cm, the density at A is 0, the density at B is 60 g/cm. **667**. $56x^6+210x^4$. **668**. $e^{x^2}(4x^2+2)$. **669**. $2\cos 2x$ **670**. $\frac{2(1-x^2)}{3(1+x^2)^2}$. **671**. $\frac{-x}{\sqrt{(a^2+x^2)^3}}$ **672**. $2\arctan x+\frac{2x}{1+x^2}$. **673**. $\frac{2}{1-x^2}+\frac{2x\arcsin x}{(1-x^2)^{3/2}}$. **674**. $\frac{1}{a}\cosh\frac{x}{a}$. **679**. $y'''=6$. **680**. $f'''(3)=4320$ **681**. $y^{V}=\frac{24}{(x+1)^5}$. **682**. $y^{VI}=-64\sin 2x$ **684**. 0; 1; 2; 2. **685**. The velocity is $v=5$; 4.997; 4.7. The acceleration, $a=0$; —0.006; —0.6. **686**. The law of motion of the point M_1 is $x=a\cos\omega t$; the velocity at time t is $-a\omega\sin\omega t$; the acceleration at time t is $-a\omega^2\cos\omega t$. Initial velocity, 0; initial acceleration: $-a\omega^2$; velocity when $x=0$ is $-a\omega$; acceleration when $x=0$ is 0. The maximum absolute value of velocity is $a\omega$; the maximum absolute value of acceleration is $a\omega^2$. **687**. $y^{(n)}=n!a^n$. **688**. a) $n!(1-x)^{-(n+1)}$, b) $(-1)^{n+1}\frac{1\cdot 3\ldots(2n-3)}{2^n x^{n-\frac{1}{2}}}$. **689**. a) $\sin\left(x+n\frac{\pi}{2}\right)$; b) $2^n\cos\left(2x+n\frac{\pi}{2}\right)$;

c) $(-3)^n e^{-3x}$; d) $(-1)^{n-1}\frac{(n-1)!}{(1+x)^n}$; e) $\frac{(-1)^{n+1}n!}{(1+x)^{n+1}}$; f) $\frac{2n!}{(1-x)^{n+1}}$; g) $2^{n-1}\sin\left[2x+(n-1)\frac{\pi}{2}\right]$; h) $\frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$. **690.** a) $x\cdot e^x+ne^x$; b) $2^{n-1}e^{-2x}\left[2(-1)^n x^2+2n(-1)^{n-1}x+\frac{n(n-1)}{2}(-1)^{n-2}\right]$; c) $(1-x^2)\times$ $\times\cos\left(x+\frac{n\pi}{2}\right)-2nx\cos\left(x+\frac{(n-1)\pi}{2}\right)-n(n-1)\cos\left(x+\frac{(n-2)\pi}{2}\right)$; d) $\frac{(-1)^{n-1}\cdot 1\cdot 3\ldots(2n-3)}{2^n x^{\frac{2n+1}{2}}}[x-(2n-1)]$; e) $\frac{(-1)^n 6(n-4)!}{x^{n-3}}$ for $n\geq 4$.

691 $y^{(n)}(0)=-(n-1)!$ **692.** a) $9t^3$; b) $2t^2+2$; c) $-\sqrt{1-t^2}$. **693.** a) $\frac{-1}{a\sin^3 t}$; b) $\frac{1}{3a\cos^4 t\sin t}$; c) $\frac{-1}{4a\sin^4\frac{t}{2}}$; d) $\frac{-1}{at\sin^3 t}$. **694.** a) 0; b) $2e^{3at}$. **695.** a) $(1+t^2)\times$ $\times(1+3t^2)$; b) $t\frac{1+t}{(1-t)^3}$. **696.** $\frac{-2e^{-t}}{(\cos t+\sin t)^3}$. **697.** $\left(\frac{d^2y}{dx^2}\right)_{t=0}=1$. **699.** $\frac{2\cot^4 t}{\sin t}$. **700.** $\frac{4e^{2t}(2\sin t-\cos t)}{(\sin t+\cos t)^5}$. **701.** $-6e^{3t}(1+3t+t^2)$. **702.** $m^n t^m$. **703.** $\frac{d^2x}{dy^2}=\frac{-f''(x)}{[f'(x)]^3}$; $\frac{d^3x}{dy^3}=\frac{3[f''(x)]^2-f'(x)f'''(x)}{[f'(x)]^5}$. **705.** $-\frac{p^2}{y^3}$. **706.** $-\frac{b^4}{a^2y^3}$. **707.** $-\frac{2y^2+2}{y^5}$. **708.** $\frac{d^2y}{dx^2}=\frac{y}{(1-y)^3}$; $\frac{d^2x}{dy^2}=\frac{1}{y^2}$. **709.** $\frac{111}{256}$. **710.** $-\frac{1}{16}$. **711.** a) $\frac{1}{3}$; b) $-\frac{3a^2x}{y^5}$. **712.** $\Delta y=0.009001$; $dy=0.009$. **713.** $d(1-x^3)=1$ when $x=1$ and $\Delta x=-\frac{1}{3}$. **714.** $dS=2x\,\Delta x$, $\Delta S=2x\,\Delta x+(\Delta x)^2$. **717.** For $x=0$. **718.** No. **719.** $dy=-\frac{\pi}{72}\approx-0.0436$. **720.** $dy=\frac{1}{2700}\approx 0.00037$. **721.** $dy=\frac{\pi}{45}\approx 0.0698$. **722.** $\frac{-m\,dx}{x^{m+1}}$. **723.** $\frac{dx}{(1-x)^2}$. **724.** $\frac{dx}{\sqrt{a^2-x^2}}$. **725.** $\frac{a\,dx}{x^2+a^2}$. **726.** $-2xe^{-x^2}dx$. **727.** $\ln x\,dx$. **728.** $\frac{-2dx}{1-x^2}$. **729.** $-\frac{1+\cos\varphi}{\sin^2\varphi}d\varphi$. **730.** $-\frac{e^t dt}{1+e^{2t}}$. **732.** $-\frac{10x+8y}{7x+5y}dx$. **733.** $\frac{-ye^{-\frac{x}{y}}dx}{y^2-xe^{-\frac{x}{y}}}=\frac{y}{x-y}dx$. **734** $\frac{x+y}{x-y}dx$. **735.** $\frac{12}{11}dx$. **737.** a) 0.485; b) 0.965; c) 1.2; d) −0.045; e) $\frac{\pi}{4}+0.025\approx 0.81$. **738.** 565 cm³. **739.** $\sqrt{5}\approx 2.25$; $\sqrt{17}\approx 4.13$; $\sqrt{70}\approx 8.38$; $\sqrt{640}\approx 25.3$. **740.** $\sqrt[3]{10}\approx 2.16$; $\sqrt[3]{70}\approx 4.13$; $\sqrt[3]{200}\approx 5.85$. **741.** a) 5; b) 1.1; c) 0.93; d) 0.9. **742.** 1.0019. **743.** 0.57. **744.** 2.03. **748.** $\frac{-(dx)^2}{(1-x^2)^{3/2}}$. **749.** $\frac{-x(dx)^2}{(1-x^2)^{3/2}}$. **750.** $\left(-\sin x\ln x+\frac{2\cos x}{x}-\frac{\sin x}{x^2}\right)(dx)^2$. **751.** $\frac{2\ln x-3}{x^3}(dx)^2$. **752.** $-e^{-x}\times$ $\times(x^2-6x+6)(dx)^3$. **753.** $\frac{384(dx)^4}{(2-x)^5}$. **754.** $3\cdot 2^n\sin\left(2x+5+\frac{n\pi}{2}\right)(dx)^n$.

755. $e^{x\cos\alpha}\sin(x\sin\alpha+n\alpha)\cdot(dx)^n$. **757.** No, since $f'(2)$ does not exist. **758.** No. The point $x=\frac{\pi}{2}$ is a discontinuity of the function. **762.** $\xi=0$. **763.** (2, 4). **765.** a) $\xi=\frac{14}{9}$; b) $\xi=\frac{\pi}{4}$. **768.** $\ln x=(x-1)-\frac{1}{2}(x-1)^2+\frac{2(x-1)^3}{3!\,\xi^3}$, where $\xi=1+\theta(x-1)$, $0<\theta<1$. **769.** $\sin x=x-\frac{x^3}{3!}+\frac{x^4}{4!}\sin\xi_1$, where $\xi_1=\theta_1 x$, $0<\theta_1<1$; $\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^6}{6!}\sin\xi_2$, where $\xi_2=\theta_2 x$, $0<\theta_2<1$. **770.** $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^{n-1}}{(n-1)!}+\frac{x^n}{n!}e^{\xi}$, where $\xi=\theta x$, $0<\theta<1$. **772.** Error: a) $\frac{1}{16}\frac{x^3}{(1+\xi)^{5/2}}$; b) $\frac{5}{81}\frac{x^3}{(1+\xi)^{8/3}}$; in both cases $\xi=\theta x$; $0<\theta<1$. **773.** The error is less than $\frac{3}{5!}=\frac{1}{40}$. **775. Solution.** We have $\sqrt{\frac{a+x}{a-x}}=\left(1+\frac{x}{a}\right)^{\frac{1}{2}}\left(1-\frac{x}{a}\right)^{\frac{1}{2}}$. Expanding both factors in powers of x, we get: $\left(1+\frac{x}{a}\right)^{\frac{1}{2}}\approx 1+\frac{1}{2}\frac{x}{a}-\frac{1}{8}\frac{x^2}{a^2}$; $\left(1-\frac{x}{a}\right)^{-\frac{1}{2}}\approx 1+\frac{1}{2}\frac{x}{a}+\frac{3}{8}\frac{x^2}{a^2}$. Multiplying, we will have: $\sqrt{\frac{a+x}{a-x}}\approx 1+\frac{x}{a}+\frac{x^2}{2a^2}$. Then, expanding $e^{\frac{x}{a}}$ in powers of $\frac{x}{a}$, we get the same polynomial $e^{\frac{x}{a}}\approx 1+\frac{x}{a}+\frac{x^2}{2a^2}$. **777.** $-\frac{1}{3}$. **778** ∞ **779.** 1 **780.** 3. **781.** $\frac{1}{2}$ **782.** 5. **783.** ∞. **784.** 0. **785.** $\frac{\pi^2}{2}$. **786.** 1. **788.** $\frac{2}{\pi}$. **789.** 1. **790.** 0. **791.** a. **792.** ∞ for $n>1$; a for $n=1$; 0 for $n<1$. **793.** 0. **795.** $\frac{1}{5}$. **796.** $\frac{1}{12}$ **797.** -1. **799.** 1. **800.** e^3. **801.** 1. **802** 1 **803.** 1 **804.** $\frac{1}{e}$. **805.** $\frac{1}{e}$. **806.** $\frac{1}{e}$. **807.** 1. **808.** 1. **810. Hint.** Find $\lim\limits_{\alpha\to 0}\frac{S}{\frac{2}{3}bh}$, where $S=\frac{R^2}{2}(\alpha-\sin\alpha)$ is the exact expression for the area of the segment (R is the radius of the corresponding circle).

Chapter III

811. $(-\infty, -2)$, increases; $(-2, \infty)$, decreases. **812.** $(-\infty, 2)$, decreases; $(2, \infty)$, increases. **813.** $(-\infty, \infty)$, increases. **814.** $(-\infty, 0)$ and $(2, \infty)$, increases; $(0, 2)$, decreases **815.** $(-\infty, 2)$ and $(2, \infty)$, decreases. **816.** $(-\infty, 1)$, increases; $(1, \infty)$, decreases. **817.** $(-\infty, -2)$, $(-2, 8)$ and $(8, \infty)$, decreases. **818.** $(0, 1)$, decreases; $(1, \infty)$, increases. **819.** $(-\infty, -1)$ and $(1, \infty)$, increases; $(-1, 1)$, decreases **820.** $(-\infty, \infty)$, increases **821.** $\left(0, \frac{1}{e}\right)$, decreases; $\left(\frac{1}{e}, \infty\right)$, increases. **822.** $(-2, 0)$, increases. **823.** $(-\infty, 2)$, decreases;

(2, ∞), increases. **824.** $(-\infty, a)$ and (a, ∞), decreases. **825.** $(-\infty, 0)$ and (0, 1), decreases; $(1, \infty)$, increases **827.** $y_{max}=\frac{9}{4}$ when $x=\frac{1}{2}$. **828.** No extremum. **830.** $y_{min}=0$ when $x=0$; $y_{min}=0$ when $x=12$; $y_{max}=1296$ when $x=6$. **831.** $y_{min}\approx -0.76$ when $x\approx 0.23$; $y_{max}=0$ when $x=1$; $y_{min}\approx -0.05$ when $x\approx 1.43$. No extremum when $x=2$. **832.** No extremum. **833.** $y_{max}=-2$ when $x=0$; $y_{min}=2$ when $x=2$ **834.** $y_{max}=\frac{9}{16}$ when $x=3.2$. **835.** $y_{max}=-3\sqrt{3}$ when $x=-\frac{2}{\sqrt{3}}$; $y_{min}=3\sqrt{3}$ when $x=\frac{2}{\sqrt{3}}$ **836.** $y_{max}=\sqrt{2}$ when $x=0$ **837.** $y_{max}=-\sqrt{3}$ when $x=-2\sqrt{3}$; $y_{min}=\sqrt{3}$ when $x=2\sqrt{3}$. **838.** $y_{min}=0$ when $x=\pm 1$; $y_{max}=1$ when $x=0$ **839.** $y_{min}=-\frac{3}{2}\sqrt{3}$ when $x=\left(k-\frac{1}{6}\right)\pi$; $y_{max}=\frac{3}{2}\sqrt{3}$ when $x=\left(k+\frac{1}{6}\pi\right)$ $(k=0, \pm 1, \pm 2, \ldots)$. **840.** $y_{max}=5$ when $x=12k\pi$; $y_{max}=5\cos\frac{2\pi}{5}$ when $x=12\left(k\pm\frac{2}{5}\right)\pi$; $y_{min}=-5\cos\frac{\pi}{5}$ when $x=12\left(k\pm\frac{1}{5}\right)\pi$; $y_{min}=1$ when $x=6(2k+1)\pi$ $(k=0, \pm 1, \pm 2, \ldots)$. **841.** $y_{min}=0$ when $x=0$. **842.** $y_{min}=-\frac{1}{e}$ when $x=\frac{1}{e}$. **843.** $y_{max}=\frac{4}{e^2}$ when $x=\frac{1}{e^2}$; $y_{min}=0$ when $x=1$ **844.** $y_{min}=1$ when $x=0$ **845.** $y_{min}=-\frac{1}{e}$ when $x=-1$. **846.** $y_{min}=0$ when $x=0$; $y_{max}=\frac{4}{e^2}$ when $x=2$ **847.** $y_{min}=e$ when $x=1$. **848.** No extremum. **849.** Smallest value is $m=-\frac{1}{2}$ for $x=-1$; greatest value, $M=\frac{1}{2}$ when $x=1$. **850.** $m=0$ when $x=0$ and $x=10$; $M=5$ for $x=5$. **851.** $m=\frac{1}{2}$ when $x=(2k+1)\frac{\pi}{4}$; $M=1$ for $x=\frac{k\pi}{2}$ $(k=0, \pm 1, \pm 2, \ldots)$. **852.** $m=0$ when $x=1$; $M=\pi$ when $x=-1$. **853.** $m=-1$ when $x=-1$; $M=27$ when $x=3$. **854.** a) $m=-6$ when $x=1$; $M=266$ when $x=5$; b) $m=-1579$ when $x=-10$; $M=3745$ when $x=12$. **856.** $p=-2$, $q=4$. **861.** Each of the terms must be equal to $\frac{a}{2}$ **862.** The rectangle must be a square with side $\frac{l}{4}$. **863.** Isosceles. **864.** The side adjoining the wall must be twice the other side **865.** The side of the cut-out square must be equal to $\frac{a}{6}$. **866.** The altitude must be half the base. **867.** That whose altitude is equal to the diameter of the base **868.** Altitude of the cylinder, $\frac{2R}{\sqrt{3}}$; radius of its base $R\sqrt{\frac{2}{3}}$, where R is the radius of the given sphere. **869.** Altitude of the cylinder, $R\sqrt{2}$ where R is the radius of the given sphere. **870.** Altitude of the cone, $\frac{4}{3}R$

where R is the radius of the given sphere. **871.** Altitude of the cone, $\frac{4}{3}R$, where R is the radius of the given sphere. **872.** Radius of the base of the cone $\frac{3}{2}r$, where r is the radius of the base of the given cylinder. **873.** That whose altitude is twice the diameter of the sphere. **874.** $\varphi=\pi$, that is, the cross-section of the channel is a semicircle. **875.** The central angle of the sector is $2\pi\sqrt{\frac{2}{3}}$. **876.** The altitude of the cylindrical part must be zero; that is, the vessel should be in the shape of a hemisphere. **877.** $h=\left(l^{\frac{2}{3}}-d^{\frac{2}{3}}\right)^{\frac{3}{2}}$. **878.** $\frac{x}{2x_0}+\frac{y}{2y_0}=1$. **879.** The sides of the rectangle are $a\sqrt{2}$ and $b\sqrt{2}$, where a and b are the respective semiaxes of the ellipse. **880.** The coordinates of the vertices of the rectangle which lie on the parabola $\left(\frac{2}{3}a;\ \pm 2\sqrt{\frac{pa}{3}}\right)$. **881.** $\left(\pm\frac{1}{\sqrt{3}},\ \frac{3}{4}\right)$. **882.** The angle is equal to the greatest of the numbers $\operatorname{arc\,cos}\frac{1}{k}$ and $\operatorname{arc\,tan}\frac{h}{d}$. **883.** $AM=a\frac{\sqrt[3]{p}}{\sqrt[3]{p}+\sqrt[3]{q}}$. **884.** $\frac{r}{\sqrt{2}}$. **885.** a) $x=y=\frac{d}{\sqrt{2}}$; b) $x=\frac{d}{\sqrt{3}}$; $y=d\sqrt{\frac{2}{3}}$. **886.** $x=\sqrt{\frac{2aQ}{q}}$; $P_{\min}=\sqrt{2aqQ}$. **887.** $\sqrt{Mm}$. **Hint.** For a completely elastic impact of two spheres, the velocity imparted to the stationary sphere of mass m_1 after impact with a sphere of mass m_2 moving with velocity v is equal to $\frac{2m_2v}{m_1+m_2}$. **888.** $n=\sqrt{\frac{NR}{r}}$ (if this number is not an integer or is not a divisor of N, we take the closest integer which is a divisor of N). Since the internal resistance of the battery is $\frac{n^2r}{N}$, the physical meaning of the solution obtained is as follows: the internal resistance of the battery must be as close as possible to the external resistance. **889.** $y=\frac{2}{3}h$. **891.** $(-\infty,\ 2)$, concave down; $(2,\ \infty)$, concave up; $M(2,\ 12)$, point of inflection. **892.** $(-\infty,\ \infty)$, concave up. **893.** $(-\infty,\ -3)$, concave down, $(-3,\ \infty)$, concave up; no points of inflection. **894.** $(-\infty,\ -6)$ and $(0,\ 6)$, concave up; $(-6,\ 0)$ and $(6,\ \infty)$, concave down; points of inflection $M_1\left(-6,\ -\frac{9}{2}\right)$, $O(0,\ 0)$, $M_2\left(6,\ \frac{9}{2}\right)$. **895.** $(-\infty,\ -\sqrt{3})$ and $(0,\ \sqrt{3})$, concave up; $(-\sqrt{3},\ 0)$ and $(\sqrt{3},\ \infty)$, concave down; points of inflection $M_{1,2}(\pm\sqrt{3},\ 0)$ and $O(0,\ 0)$. **896.** $\left((4k+1)\frac{\pi}{2},\ (4k+3)\frac{\pi}{2}\right)$, concave up; $\left((4k+3)\frac{\pi}{2},\ (4k+5)\frac{\pi}{2}\right)$, concave down $(k=0,\ \pm 1,\ \pm 2,\ \ldots)$; points of inflection, $\left((2k+1)\frac{\pi}{2},\ 0\right)$. **897.** $(2k\pi,\ (2k+1)\pi)$, concave up; $((2k-1)\pi,\ 2k\pi)$, concave down $(k=0,\ \pm 1,\ \pm 2,\ \ldots)$; the abscissas of the points of inflection are equal to $x=k\pi$. **898.** $\left(0,\ \frac{1}{\sqrt{e^3}}\right)$, concave down; $\left(\frac{1}{\sqrt{e^3}},\ \infty\right)$, concave up; $M\left(\frac{1}{\sqrt{e^3}},\ -\frac{3}{2e^3}\right)$ is a point of inflection. **899.** $(-\infty,\ 0)$, concave up; $(0,\ \infty)$, concave down; $O(0,\ 0)$ is a point of inflection. **900.** $(-\infty,\ -3)$ and $(-1,\ \infty)$, concave up; $(-3,\ -1)$, concave down; points of inflection are $M_1\left(-3,\ \frac{10}{e^3}\right)$ and $M_2\left(-1,\ \frac{2}{e}\right)$. **901.** $x=2$, $y=0$. **902.** $x=1$, $x=3$, $y=0$ **903.** $x=\pm 2$, $y=1$. **904.** $y=x$. **905.** $y=-x$, left, $y=x$, right. **906.** $y=-1$, left, $y=1$, right **907.** $x=\pm 1$, $y=-x$, left, $y=x$, right **908.** $y=-2$, left, $y=2x-2$, right. **909.** $y=2$ **910.** $x=0$, $y=1$, left, $y=0$, right. **911.** $x=0$, $y=1$. **912.** $y=0$. **913.** $x=-1$. **914.** $y=x-\pi$, left; $y=x+\pi$, right. **915.** $y=a$. **916.** $y_{max}=0$ when $x=0$; $y_{min}=-4$ when $x=2$; point of inflection, $M_1(1,\ -2)$. **917.** $y_{max}=1$ when $x=\pm\sqrt{3}$; $y_{min}=0$ when $x=0$; points of inflection $M_{1,2}\left(\pm 1,\ \frac{5}{9}\right)$ **918.** $y_{max}=4$ when $x=-1$; $y_{min}=0$ when $x=1$, point of inflection, $M_1(0,\ 2)$. **919.** $y_{max}=8$ when $x=-2$, $y_{min}=0$ when $x=2$; point of inflection, $M(0,\ 4)$. **920.** $y_{min}=-1$ when $x=0$; points of inflection $M_{1,2}(\pm\sqrt{5},\ 0)$ and $M_{3,4}\left(\pm 1,\ -\frac{64}{125}\right)$. **921.** $y_{max}=-2$ when $x=0$; $y_{min}=2$ when $x=2$; asymptotes, $x=1$, $y=x-1$. **922.** Points of inflection $M_{1,2}(\pm 1,\ \mp 2)$; asymptote $x=0$. **923.** $y_{max}=-4$ when $x=-1$; $y_{min}=4$ when $x=1$; asymptote, $x=0$. **924.** $y_{min}=3$ when $x=1$; point of inflection, $M(-\sqrt[3]{2},\ 0)$; asymptote, $x=0$. **925.** $y_{max}=\frac{1}{3}$ when $x=0$, points of inflection, $M_{1,2}\left(\pm 1,\ \frac{1}{4}\right)$; asymptote, $y=0$ **926.** $y_{max}=-2$ when $x=0$; asymptotes, $x=\pm 2$ and $y=0$. **927.** $y_{min}=-1$ when $x=-1$; $y_{max}=1$ when $x=1$; points of inflection, $O(0,\ 0)$ and $M_{1,2}\left(\pm 2\sqrt{3},\ \pm\frac{\sqrt{3}}{2}\right)$; asymptote, $y=0$ **928.** $y_{max}=1$ when $x=4$; point of inflection, $M\left(5,\ \frac{8}{9}\right)$; asymptotes, $x=2$ and $y=0$. **929.** Point of inflection, $O(0,\ 0)$; asymptotes, $x=\pm 2$ and $y=0$. **930.** $y_{max}=-\frac{27}{16}$ when $x=\frac{8}{3}$; asymptotes, $x=0$, $x=4$ and $y=0$ **931.** $y_{max}=-4$ when $x=-1$; $y_{min}=4$ when $x=1$; asymptotes, $x=0$ and $y=3x$ **932.** $A(0,\ 2)$ and $B(4,\ 2)$ are end-points; $y_{max}=2\sqrt{2}$ when $x=2$ **933.** $A(-8,\ -4)$ and $B(8,\ 4)$ are end-points. Point of inflection, $O(0,\ 0)$. **934.** End-point, $A(-3,\ 0)$; $y_{min}=-2$ when $x=-2$. **935.** End-points, $A(-\sqrt{3},\ 0)$, $O(0,\ 0)$ and $B(\sqrt{3},\ 0)$; $y_{max}=\sqrt{2}$ when $x=-1$; point of inflection, $M\left(\sqrt{3+2\sqrt{3}},\ \sqrt{6\sqrt{1+\frac{2}{\sqrt{3}}}}\right)$. **936.** $y_{max}=1$ when $x=0$, points of inflection, $M_{1,2}(\pm 1,\ 0)$. **937.** Points of inflection, $M_1(0,\ 1)$ and $M_2(1,\ 0)$; asymptote, $y=-x$. **938.** $y_{max}=0$ when $x=-1$; $y_{min}=-1$ (when $x=0$) **939.** $y_{max}=2$ when $x=0$; points of inflection, $M_{1,2}(\pm 1,\ \sqrt[3]{2})$; asymptote, $y=0$. **940.** $y_{min}=-4$ when $x=-4$; $y_{max}=4$ when $x=4$; point of inflection, $O(0,\ 0)$; asymptote, $y=0$. **941.** $y_{min}=\sqrt[3]{4}$ when $x=2$, $y_{min}=\sqrt[3]{4}$ when $x=4$; $y_{max}=2$ when $x=3$. **942.** $y_{min}=2$ when $x=0$; asymptote, $x=\pm 2$. **943.** Asymptotes, $x=\pm 2$ and $y=0$. **944.** $y_{min}=\frac{\sqrt{3}}{\sqrt[3]{2}}$ when $x=\sqrt{3}$;

$y_{max}=-\frac{\sqrt{3}}{\sqrt[3]{2}}$ when $x=-3$; points of inflection, $M_1\left(-3, -\frac{3}{2}\right)$, $O(0, 0)$ and $M_2\left(3, \frac{3}{2}\right)$; asymptotes, $x=\pm 1$ **945.** $y_{min}=\frac{3}{\sqrt[3]{2}}$ when $x=6$; point of inflection, $M\left(12, \frac{12}{\sqrt[3]{100}}\right)$; asymptote, $x=2$ **946.** $y_{max}=\frac{1}{e}$ when $x=1$; point of inflection, $M\left(2, \frac{2}{e^2}\right)$; asymptote, $y=0$. **947.** Points of inflection, $M_1\left(-3a, \frac{10a}{e^3}\right)$ and $M_2\left(-a, \frac{2a}{e}\right)$; asymptote, $y=0$. **948.** $y_{max}=e^2$ when $x=4$; points of inflection, $M_{1,2}\left(\frac{8\pm 2\sqrt{2}}{2}, e^{\frac{3}{2}}\right)$; asymptote, $y=0$. **949.** $y_{max}=2$ when $x=0$; points of inflection, $M_{1,2}\left(\pm 1, \frac{3}{e}\right)$. **950.** $y_{max}=1$ when $x=\pm 1$; $y_{min}=0$ when $x=0$. **951.** $y_{max}=0.74$ when $x=e^2\approx 7\ 39$; point of inflection, $M(e^{\pi/3}\approx 14.39,\ 0.70)$; asymptotes, $x=0$ and $y=0$. **952.** $y_{min}=-\frac{a^2}{4e}$ when $x=\frac{a}{\sqrt{e}}$, point of inflection, $M\left(\frac{a}{\sqrt{e^3}}, -\frac{3a^2}{4e^2}\right)$. **953.** $y_{min}=e$ when $x=e$; point of inflection, $M\left(e^2, \frac{e^2}{2}\right)$; asymptote, $x=1$; $y\to 0$ when $x\to 0$. **954.** $y_{max}=\frac{4}{e^2}\approx 0.54$ when $x=\frac{1}{e^2}-1\approx -0.86$; $y_{min}=0$ when $x=0$; point of inflection, $M\left(\frac{1}{e}-1\approx -0.63;\ \frac{1}{e}\approx 0.37\right)$; $y\to 0$ as $x\to -1+0$ (limiting end-point). **955.** $y_{min}=1$ when $x=\pm\sqrt{2}$; points of inflection, $M_{1,2}(\pm 1.89,\ 1.33)$; asymptotes, $x=\pm 1$. **956.** Asymptote, $y=0$. **957.** Asymptotes, $y=0$ (when $x\to +\infty$) and $y=-x$ (as $x\to -\infty$). **958.** Asymptotes, $x=-\frac{1}{e}$, $x=0$, $y=1$; the function is not defined on the interval $\left[-\frac{1}{e}, 0\right]$. **959.** Periodic function with period 2π. $y_{min}=-\sqrt{2}$ when $x=\frac{5}{4}\pi+2k\pi$; $y_{max}=\sqrt{2}$ when $x=\frac{\pi}{4}+2k\pi$ $(k=0, \pm 1, \pm 2, \ldots)$; points of inflection, $M_k\left(\frac{3}{4}\pi+k\pi, 0\right)$. **960.** Periodic function with period 2π. $y_{min}=-\frac{3}{4}\sqrt{3}$ when $x=\frac{5}{3}\pi+2k\pi$; $y_{max}=\frac{3}{4}\sqrt{3}$ when $x=\frac{\pi}{3}+2k\pi$ $(k=0, \pm 1, \pm 2, \ldots)$; points of inflection, $M_k(k\pi, 0)$ and $N_k\left(\arccos\left(-\frac{1}{4}\right)+2k\pi, \frac{3}{16}\sqrt{15}\right)$. **961.** Periodic function with period 2π. On the interval $[-\pi, \pi]$, $y_{max}=\frac{1}{4}$ when $x=\pm\frac{\pi}{3}$; $y_{min}=-2$ when $x=\pm\pi$; $y_{min}=0$ when $x=0$; points of inflection, $M_{1,2}(\pm 0.57,\ 0.13)$ and $M_{3,4}(\pm 2\ 20,\ -0.95)$. **962.** Odd periodic function with period 2π. On interval $[0, 2\pi]$, $y_{max}=1$ when $x=0$; $y_{min}=0.71$, when $x=\frac{\pi}{4}$; $y_{max}=1$ when

$x=\frac{\pi}{2}$; $y_{min}=-1$ when $x=\pi$; $y_{max}=-0.71$ when $x=\frac{5}{4}\pi$; $y_{min}=-1$ when $x=\frac{3}{2}\pi$; $y_{max}=1$ when $x=2\pi$; points of inflection, $M_1(0.36,\ 0.86)$; $M_2(1.21,\ 0.86)$; $M_3(2.36,\ 0)$; $M_4(3.51,\ -0.86)$; $M_5(4.35,\ -0.86)$; $M_6(5.50,\ 0)$. **963.** Periodic function with period 2π. $y_{min}=\frac{\sqrt{2}}{2}$ when $x=\frac{\pi}{4}+2k\pi$; $y_{max}=-\frac{\sqrt{2}}{2}$ when $x=-\frac{3}{4}\pi+2k\pi$ $(k=0,\ \pm 1,\ \pm 2,\ \ldots)$; asymptotes, $x=\frac{3}{4}\pi+k\pi$ **964.** Periodic function with period π; points of inflection, $M_k\left(\frac{\pi}{4}+k\pi,\ \frac{\sqrt{2}}{2}\right)$ $(k=0,\ \pm 1,\ \pm 2,\ \ldots)$; asymptotes, $x=\frac{3}{4}\pi+k\pi$. **965.** Even periodic function with period 2π On the interval $[0,\ \pi$ $y_{max}=\frac{4}{3\sqrt{3}}$ when $x=\arccos\frac{1}{\sqrt{3}}$; $y_{max}=0$ when $x=\pi$; $y_{min}=-\frac{4}{3\sqrt{3}}$ when $x=\arccos\left(-\frac{1}{\sqrt{3}}\right)$; $y_{min}=0$ when $x=0$; points of inflection, $M_1\left(\frac{\pi}{2},\ 0\right)$; $M_2\left(\arcsin\frac{\sqrt{2}}{3},\ \frac{4\sqrt{7}}{27}\right)$; $M_3\left(\pi-\arcsin\frac{\sqrt{2}}{3},\ -\frac{4\sqrt{7}}{27}\right)$. **966.** Even periodic function with period 2π. On the interval $[0,\ \pi]$ $y_{max}=1$ when $x=0$; $y_{max}=\frac{2}{3\sqrt{6}}$ when $x=\arccos\left(-\frac{1}{\sqrt{6}}\right)$; $y_{min}=-\frac{2}{3\sqrt{6}}$ when $x=\arccos\frac{1}{\sqrt{6}}$; $y_{min}=-1$ when $x=\pi$; points of inflection, $M_1\left(\frac{\pi}{2},0\right)$; $M_2\left(\arccos\sqrt{\frac{13}{18}},\ \frac{4}{9}\sqrt{\frac{13}{18}}\right)$; $M_3\left(\arccos\left(-\sqrt{\frac{13}{18}}\right),\ -\frac{4}{9}\sqrt{\frac{13}{18}}\right)$.
967. Odd function. Points of inflection, $M_k(k\pi,\ k\pi)$ $(k=0,\ \pm 1,\ \pm 2,\ \ldots)$. **968.** Even function. End-points, $A_{1,2}(\pm 2\ 83,\ -1\ 57)$ $y_{max}=1\ 57$ when $x=0$ (cusp); points of inflection, $M_{1,2}(\pm 1.54,\ -0.34)$. **969.** Odd function. Limiting points of graph $(-1,\ -\infty)$ and $(1,\ +\infty)$. Point of inflection, $O(0,\ 0)$; asymptotes, $x=\pm 1$. **970.** Odd function. $y_{max}=\frac{\pi}{2}-1+2k\pi$ when $x=\frac{\pi}{4}+k\pi$; $y_{min}=\frac{3}{2}\pi+1+2k\pi$ when $x=\frac{3}{4}\pi+k\pi$; points of inflection, $M_k(k\pi,\ 2k\pi)$; asymptotes, $x=\frac{2k+1}{2}\pi$ $(k=0,\ \pm 1,\ \pm 2,\ \ldots)$. **971.** Even function. $y_{min}=0$ when $x=0$; asymptotes, $y=-\frac{\pi}{2}x-1$ (as $x\to-\infty$) and $y=\frac{\pi}{2}x-1$ (as $x\to+\infty$). **972.** $y_{min}=0$ when $x=0$ (node); asymptote, $y=1$. **973.** $y_{min}=1+\frac{\pi}{2}$ when $x=1$; $y_{max}=\frac{3\pi}{2}-1$ when $x=-1$; point of inflection (centre of symmetry) $(0,\ \pi)$; asymptotes, $y=x+2\pi$ (left) and $y=x$ (right). **974.** Odd function. $y_{min}=1.285$ when $x=1$; $y_{max}=1.856$ when $x=-1$; point of inflection, $M\left(0,\ \frac{\pi}{2}\right)$; asymptotes, $y=\frac{x}{2}+\pi$ (when $x\to-\infty$) and $y=\frac{x}{2}$ (as $x\to+\infty$). **975.** Asymptotes, $x=0$ and $y=x-\ln 2$.

976. $y_{min}=1.32$ when $x=\pm 1$; asymptote, $x=0$. **977.** Periodic function with period 2π. $y_{min}=\frac{1}{6}$ when $x=\frac{3}{2}\pi+2k\pi$; $y_{max}=e$ when $x=\frac{\pi}{2}+2k\pi$ ($k=0, \pm 1, \pm 2, \ldots$); points of inflection, $M_k\left(\arcsin\frac{\sqrt{5}-1}{2}+2k\pi,\ e^{\frac{\sqrt{5}-1}{2}}\right)$ and $N_k\left(-\arcsin\frac{\sqrt{5}-1}{2}+(2k+1)\pi,\ e^{\frac{\sqrt{5}+1}{2}}\right)$. **978.** End-points, $A(0, 1)$ and $B(1, 4.81)$. Point of inflection, $M(0.28, 1.74)$. **979.** Points of inflection, $M(0.5, 1.59)$; asymptotes, $y=0.21$ (as $x\to-\infty$) and $y=4.81$ (as $x\to+\infty$). **980.** The domain of definition of the function is the set of intervals $(2k\pi, 2k\pi+\pi)$, where $k=0, \pm 1, \pm 2, \ldots$ Periodic function with period 2π. $y_{max}=0$ when $x=\frac{\pi}{2}+2k\pi$ ($k=0, \pm 1, \pm 2, \ldots$); asymptotes, $x=k\pi$. **981.** The domain of definition is the set of intervals $\left[\left(2k-\frac{1}{2}\right)\pi, \left(2k+\frac{1}{2}\right)\pi\right]$, where k is an integer. Periodic function with period 2π. Points of inflection, $M_k(2k\pi, 0)$ ($k=0, \pm 1, \pm 2, \ldots$); asymptotes, $x=\pm\frac{\pi}{2}+2k\pi$. **982.** Domain of definition, $x>0$; monotonic increasing function; asymptote, $x=0$. **983.** Domain of definition, $|x-2k\pi|<\frac{\pi}{2}$ ($k=0, \pm 1, \pm 2, \ldots$). Periodic function with period 2π $y_{min}=1$ when $x=2k\pi$ ($k=0, \pm 1, \pm 2, \ldots$); asymptotes, $x=\frac{\pi}{2}+k\pi$. **984.** Asymptote, $y=1.57$; $y\to-1.57$ as $x\to 0$ (limiting end-point). **985.** End-points, $A_{1,2}(\pm 1.31, 1\ 57)$; $y_{min}=0$ when $x=0$. **986.** $y_{min}=\left(\frac{1}{e}\right)^{\frac{1}{e}}\approx 0.69$ when $x=\frac{1}{e}\approx 0.37$; $y\to 1$ as $x\to+0$ **987.** Limiting end-point, $A(+0, 0)$; $y_{max}=e^{\frac{1}{e}}\approx 1.44$ when $x=e\approx 2.72$; asymptote, $y=1$; point of inflection, $M_1(0.58, 0.12)$ and $M_2(4\ 35, 1\ 40)$. **988.** $x_{min}=-1$ when $t=1$ ($y=3$); $y_{min}=-1$ when $t=-1$ ($x=3$) **989.** To obtain the graph it is sufficient to vary t from 0 to 2π. $x_{min}=-a$ when $t=\pi$ ($y=0$); $x_{max}=a$ when $t=0$ ($y=0$); $y_{min}=-a$ (cusp) when $t=+\frac{3\pi}{2}$ ($x=0$); $y_{max}=+a$ (cusp) when $t=\frac{\pi}{2}$ ($x=0$); points of inflection when $t=\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ $\left(x=\pm\frac{a}{2\sqrt{2}}, y=\pm\frac{a}{\sqrt{2}}\right)$.

990. $x_{min}=-\frac{1}{e}$ when $t=-1$ ($y=-e$); $y_{max}=\frac{1}{e}$ when $t=1$ ($x=e$); points of inflection when $t=-\sqrt{2}$, i.e., $\left(-\frac{\sqrt{2}}{e^{\sqrt{2}}}, -\sqrt{2}e^{\sqrt{2}}\right)$ and when $t=\sqrt{2}$, i.e., $\left(\sqrt{2}\,e^{\sqrt{2}}, \frac{\sqrt{2}}{e^{\sqrt{2}}}\right)$; asymptotes, $x=0$ and $y=0$. **991.** $x_{min}=1$ and $y_{min}=1$ when $t=0$ (cusp); asymptote, $y=2x$ when $t\to+\infty$. **992.** $y_{min}=0$ when $t=0$.

993. $ds=\frac{a}{y}\,dx$, $\cos\alpha=\frac{y}{a}$; $\sin\alpha=-\frac{x}{a}$. 994. $ds=\frac{1}{a}\sqrt{\frac{a^4-c^2x^2}{a^2-x^2}}\,dx$; $\cos\alpha=\frac{a\sqrt{a^2-x^2}}{\sqrt{a^4-c^2x^2}}$; $\sin\alpha=-\frac{bx}{\sqrt{a^2-c^2x^2}}$, where $c=\sqrt{a^2-b^2}$. 995. $ds=\frac{1}{y}\sqrt{p^2+y^2}\,dx$; $\cos\alpha=\frac{y}{\sqrt{p^2+y^2}}$; $\sin\alpha=\frac{p}{\sqrt{p^2+y^2}}$. 996. $ds=\sqrt[3]{\frac{a}{x}}\,dx$; $\cos\alpha=\sqrt[3]{\frac{x}{a}}$; $\sin\alpha=-\sqrt[3]{\frac{y}{a}}$. 997. $ds=\cosh\frac{x}{a}\,dx$; $\cos\alpha=\frac{1}{\cosh\frac{x}{a}}$; $\sin\alpha=\tanh\frac{x}{a}$. 998. $ds=2a\sin\frac{t}{2}\,dt$; $\cos\alpha=\sin\frac{t}{2}$; $\sin\alpha=\cos\frac{t}{2}$. 999. $ds=3a\sin t\cos t\,dt$; $\cos\alpha=-\cos t$; $\sin\alpha=\sin t$. 1000. $ds=a\sqrt{1+\varphi^2}\,d\varphi$; $\cos\beta=\frac{1}{\sqrt{1+\varphi^2}}$. 1001. $ds=\frac{a}{\varphi^2}\sqrt{1+\varphi^2}\,d\varphi$; $\cos\beta=-\frac{1}{\sqrt{1+\varphi^2}}$. 1002. $ds=\frac{a}{\cos^3\frac{\varphi}{2}}\,d\varphi$; $\sin\beta=\cos\frac{\varphi}{2}$. 1003. $ds=a\cos\frac{\varphi}{2}\,d\varphi$; $\sin\beta=\cos\frac{\varphi}{2}$. 1004. $ds=r\sqrt{1+(\ln a)^2}\,d\varphi$; $\sin\beta=\frac{1}{\sqrt{1+(\ln a)^2}}$. 1005. $ds=\frac{a^2}{r}\,d\varphi$; $\sin\beta=\cos 2\varphi$. 1006. $K=36$. 1007. $K=\frac{1}{3\sqrt{2}}$. 1008. $K_A=\frac{a}{b^2}$; $K_B=\frac{b}{a^2}$. 1009. $K=\frac{6}{13\sqrt{13}}$. 1010. $K=\frac{3}{a\sqrt{2}}$ at both vertices. 1011. $\left(\frac{9}{8},\ 3\right)$ and $\left(\frac{9}{8},\ -3\right)$. 1012. $\left(-\frac{\ln 2}{2},\ \frac{\sqrt{2}}{2}\right)$. 1013. $R=\left|\frac{(1+9x^4)^{3/2}}{6x}\right|$. 1014. $R=\frac{(b^4x^2+a^4y^2)^{3/2}}{a^4b^4}$. 1015. $R=\left|\frac{(y^2+1)^2}{4y}\right|$. 1016. $R=\left|\frac{3}{2}a\sin 2t\right|$. 1017. $R=|at|$. 1018. $R=|r\sqrt{1+k^2}|$. 1019. $R=\left|\frac{4}{3}a\cos\frac{\varphi}{2}\right|$. 1020. $R_{\text{least}}=|p|$. 1022. (2,2). 1023. $\left(-\frac{11}{2}a,\ \frac{16}{3}a\right)$. 1024. $(x-3)^2+\left(y-\frac{3}{2}\right)^2=\frac{1}{4}$. 1025. $(x+2)^2+(y-3)^2=8$. 1026. $pY^2=\frac{8}{27}(X-p)^3$ (semicubical parabola). 1027. $(aX)^{\frac{2}{3}}+(bY)^{\frac{2}{3}}=c^{\frac{4}{3}}$, where $c^2=a^2-b^2$.

Chapter IV

In the answers of this section the arbitrary additive constant C is omitted for the sake of brevity. 1031. $\frac{5}{7}a^2x^7$. 1032. $2x^3+4x^2+3x$. 1033. $\frac{x^4}{4}+\frac{(a+b)x^3}{3}+\frac{abx^2}{2}$. 1034. $a^2x+\frac{abx^4}{2}+\frac{b^2x^7}{7}$. 1035. $\frac{2x}{3}\sqrt{2px}$. 1036. $\frac{nx^{\frac{n-1}{n}}}{n-1}$.

1037. $\sqrt[n]{nx}$. **1038.** $a^2x-\frac{9}{5}a^{\frac{4}{3}}x^{\frac{5}{3}}+\frac{9}{7}a^{\frac{2}{3}}x^{\frac{7}{3}}-\frac{x^3}{3}$. **1039.** $\frac{2x^2\sqrt{x}}{5}+x$.
1040. $\frac{3x^4\sqrt[3]{x}}{13}-\frac{3x^2\sqrt[3]{x}}{7}-6\sqrt[3]{x}$. **1041.** $\frac{2x^{2m}\sqrt{x}}{4m+1}-\frac{4x^{m+n}\sqrt{x}}{2m+2n+1}+\frac{2x^{2n}\sqrt{x}}{4n+1}$.
1042. $2a\sqrt{ax}-4ax+4x\sqrt{ax}-2x^2+\frac{2x^3}{5\sqrt{ax}}$ **1043.** $\frac{1}{\sqrt{7}}\ \text{arc tan}\ \frac{x}{\sqrt{7}}$.
1044. $\frac{1}{2\sqrt{10}}\ln\left|\frac{x-\sqrt{10}}{x+\sqrt{10}}\right|$. **1045.** $\ln(x+\sqrt{4+x^2})$. **1046.** $\text{arc sin}\frac{x}{2\sqrt{2}}$.
1047. $\text{arc sin}\ \frac{x}{\sqrt{2}}-\ln(x+\sqrt{x^2+2})$. **1048*.** a) $\tan x-x$. **Hint.** Put $\tan^2 x=\sec^2 x-1$; b) $x-\tanh x$. **Hint.** Put $\tanh^2 x=1-\frac{1}{\cosh^2 x}$. **1049.** a) $-\cot x-x$; b) $x-\coth x$. **1050.** $\frac{(3e)^x}{\ln 3+1}$. **1051.** $a\ln\left|\frac{c}{a-x}\right|$. **Solution.** $\int\frac{a}{a-x}dx=-a\int\frac{d(a-x)}{a-x}=-a\ln|a-x|+a\ln c=a\ln\left|\frac{c}{a-x}\right|$. **1052.** $x+\ln|2x+1|$.
Solution. Dividing the numerator by the denominator, we get $\frac{2x+3}{2x+1}=1+\frac{2}{2x+1}$. Whence $\int\frac{2x+3}{2x+1}dx=\int dx+\int\frac{2\,dx}{2x+1}=x+\int\frac{d(2x+1)}{2x+1}=x+\ln|2x+1|$. **1053.** $-\frac{3}{2}x+\frac{11}{4}\ln|3+2x|$. **1054.** $\frac{x}{b}-\frac{a}{b^2}\ln|a+bx|$.
1055. $\frac{a}{\alpha}x+\frac{b\alpha-a\beta}{\alpha^2}\ln|\alpha x+\beta|$. **1056.** $\frac{x^2}{2}+x+2\ln|x-1|$. **1057.** $\frac{x^2}{2}+2x+\ln|x+3|$. **1058.** $\frac{x^4}{4}+\frac{x^3}{3}+x^2+2x+3\ln|x-1|$. **1059.** $a^2x+2ab\ln|x-a|-\frac{b^2}{x-a}$. **1060.** $\ln|x+1|+\frac{1}{x+1}$. **Hint.** $\int\frac{x\,dx}{(x+1)^2}=\int\frac{(x+1)-1}{(x+1)^2}dx=\int\frac{dx}{x+1}-\int\frac{dx}{(x+1)^2}$. **1061.** $-2b\sqrt{1-y}$. **1062.** $-\frac{2}{3b}\sqrt{(a-bx)^3}$.
1063. $\sqrt{x^2+1}$. **Solution.** $\int\frac{x\,dx}{\sqrt{x^2+1}}=\frac{1}{2}\int\frac{d(x^2+1)}{\sqrt{x^2+1}}=\sqrt{x^2+1}$. **1064.** $2\sqrt{x}+\frac{\ln^2 x}{2}$. **1065.** $\frac{1}{\sqrt{15}}\text{arc tan}\,x\sqrt{\frac{3}{5}}$. **1066.** $\frac{1}{4\sqrt{14}}\ln\left|\frac{x\sqrt{7}-2\sqrt{2}}{x\sqrt{7}+2\sqrt{2}}\right|$.
1067. $\frac{1}{2\sqrt{a^2-b^2}}\ln\left|\frac{\sqrt{a+b}+x\sqrt{a-b}}{\sqrt{a+b}-x\sqrt{a-b}}\right|$. **1068.** $x-\sqrt{2}\ \text{arc tan}\ \frac{x}{\sqrt{2}}$.
1069. $-\left(\frac{x^2}{2}+\frac{a^2}{2}\ln|a^2-x^2|\right)$ **1070.** $x-\frac{5}{2}\ln(x^2+4)+\arctan\frac{x}{2}$.
1071. $\frac{1}{2\sqrt{2}}\ln(2\sqrt{2}x+\sqrt{7+8x^2})$. **1072.** $\frac{1}{\sqrt{5}}\text{arc sin}\,x\sqrt{\frac{5}{7}}$.
1073. $\frac{1}{3}\ln|3x^2-2|-\frac{5}{2\sqrt{6}}\ln\left|\frac{x\sqrt{3}-\sqrt{2}}{x\sqrt{3}+\sqrt{2}}\right|$. **1074.** $\frac{3}{\sqrt{35}}\text{arc tan}\sqrt{\frac{5}{7}}x-$

$-\frac{1}{5}\ln(5x^2+7)$. 1075. $\frac{3}{5}\sqrt{5x^2+1}+\frac{1}{\sqrt{5}}\ln(x\sqrt{5}+\sqrt{5x^2+1})$. 1076. $\sqrt{x^2-4}+$ $+3\ln|x+\sqrt{x^2-4}|$. 1077. $\frac{1}{2}\ln|x^2-5|$. 1078. $\frac{1}{4}\ln(2x^2+3)$. 1079. $\frac{1}{2a}\ln(a^2x^2+b^2)+\frac{1}{a}\arctan\frac{ax}{b}$. 1080. $\frac{1}{2}\arcsin\frac{x^2}{a^2}$. 1081. $\frac{1}{3}\arctan x^3$. 1082. $\frac{1}{3}\ln|x^3+\sqrt{x^6-1}|$. 1083. $\frac{2}{3}\sqrt{(\arcsin x)^3}$. 1084. $\frac{\left(\arctan\frac{x}{2}\right)^2}{4}$. 1085. $\frac{1}{8}\ln(1+4x^2)-\frac{\sqrt{(\arctan 2x)^3}}{3}$ 1086. $2\sqrt{\ln(x+\sqrt{1+x^2})}$. 1087. $-\frac{a}{m}e^{-mx}$. 1088. $-\frac{1}{3\ln 4}4^{2-3x}$. 1089. e^t+e^{-t}. 1090. $\frac{a}{2}e^{\frac{2x}{a}}+2x-$ $-\frac{a}{2}e^{-\frac{2x}{a}}$. 1091. $\frac{1}{\ln a-\ln b}\left(\frac{a^x}{b^x}-\frac{b^x}{a^x}\right)-2x$. 1092. $\frac{2}{3\ln a}\sqrt{a^{3x}}+\frac{2}{\ln a\sqrt{a^x}}$. 1093. $-\frac{1}{2e^{x^2+1}}$. 1094. $\frac{1}{2\ln 7}7^{x^2}$. 1095. $-e^{\frac{1}{x}}$. 1096. $\frac{2}{\ln 5}5^{\sqrt{x}}$. 1097. $\ln|e^x-1|$. 1098. $-\frac{2}{3b}\sqrt{(a-be^x)^3}$ 1099. $\frac{3a}{4}(e^{\frac{x}{a}}+1)^{\frac{4}{3}}$. 1100. $\frac{x}{3}-$ $-\frac{1}{3\ln 2}\ln(2^x+3)$ Hint. $\frac{1}{2^x+3}\equiv\frac{1}{3}\left(1-\frac{2^x}{2^x+3}\right)$. 1101. $\frac{1}{\ln a}\arctan(a^x)$. 1102. $-\frac{1}{2b}\ln\left|\frac{1+e^{-bx}}{1-e^{-bx}}\right|$. 1103. $\arcsin e^t$ 1104. $-\frac{1}{b}\cos(a+bx)$. 1105. $\sqrt{2}\sin\frac{x}{\sqrt{2}}$ 1106. $x-\frac{1}{2a}\cos 2ax$. 1107. $2\sin\sqrt{x}$. 1108. $-\ln 10\times$ $\times\cos(\log x)$ 1109. $\frac{x}{2}-\frac{\sin 2x}{4}$. Hint. Put $\sin^2 x=\frac{1}{2}(1-\cos 2x)$. 1110. $\frac{x}{2}+$ $+\frac{\sin 2x}{4}$ Hint. See hint in 1109 1111. $\frac{1}{a}\tan(ax+b)$. 1112. $-\frac{\cot ax}{a}-x$. 1113. $a\ln\left|\tan\frac{x}{2a}\right|$. 1114. $\frac{1}{15}\ln\left|\tan\left(\frac{5x}{2}+\frac{\pi}{8}\right)\right|$. 1115. $\frac{1}{a}\ln\left|\tan\frac{ax+b}{2}\right|$. 1116. $\frac{1}{2}\tan(x^2)$. 1117. $\frac{1}{2}\cos(1-x^2)$. 1118. $x-\frac{1}{\sqrt{2}}\cot x\sqrt{2}-$ $-\sqrt{2}\ln\left|\tan\frac{x\sqrt{2}}{2}\right|$. 1119. $-\ln|\cos x|$. 1120. $\ln|\sin x|$. 1121. $(a-b)\times$ $\times\ln\left|\sin\frac{x}{a-b}\right|$. 1122. $5\ln\left|\sin\frac{x}{5}\right|$. 1123. $-2\ln|\cos\sqrt{x}|$. 1124. $\frac{1}{2}\ln\times$ $\times|\sin(x^2+1)|$. 1125. $\ln|\tan x|$. 1126. $\frac{a}{2}\sin^2\frac{x}{a}$. 1127. $\frac{\sin^4 6x}{24}$. 1128. $-\frac{1}{4a\sin^4 ax}$. 1129. $-\frac{1}{3}\ln(3+\cos 3x)$. 1130. $-\frac{1}{2}\sqrt{\cos 2x}$. 1131. $-\frac{2}{9}\sqrt{(1+3\cos^2 x)^3}$. 1132. $\frac{3}{4}\tan^4\frac{x}{3}$. 1133. $\frac{2}{3}\sqrt{\tan^3 x}$. 1134. $-\frac{3\cot^{\frac{5}{3}}x}{5}$. 1135. $\frac{1}{3}\left(\tan 3x+\frac{1}{\cos 3x}\right)$. 1136. $\frac{1}{a}\left(\ln\left|\tan\frac{ax}{2}\right|+2\sin ax\right)$.

1137. $\frac{1}{3a}\ln|b-a\cot 3x|$. **1138.** $\frac{2}{5}\cosh 5x-\frac{3}{5}\sinh 5x$. **1139.** $-\frac{x}{2}+\frac{1}{4}\sinh 2x$. **1140.** $\ln\left|\tanh\frac{x}{2}\right|$. **1141.** $2\arctan e^x$. **1142.** $\ln|\tanh x|$. **1143.** $\ln\cosh x$. **1144.** $\ln|\sinh x|$. **1145.** $-\frac{5}{12}\sqrt[5]{(5-x^2)^6}$. **1146.** $\frac{1}{4}\ln\left|x^4-4x+1\right|$. **1147.** $\frac{1}{4\sqrt{5}}\times\arctan\frac{x^4}{\sqrt{5}}$. **1148.** $-\frac{1}{2}e^{-x^2}$. **1149.** $\sqrt{\frac{3}{2}}\arctan\left(x\sqrt{\frac{3}{2}}\right)-\frac{1}{\sqrt{3}}\ln(x\sqrt{3}+\sqrt{2+3x^2})$. **1150.** $\frac{x^3}{3}-\frac{x^2}{2}+x-2\ln|x+1|$. **1151.** $-\frac{2}{\sqrt{e^x}}$. **1152.** $\ln|x+\cos x|$. **1153.** $\frac{1}{3}\left(\ln|\sec 3x+\tan 3x|+\frac{1}{\sin 3x}\right)$. **1154.** $-\frac{1}{\ln x}$. **1155.** $\ln|\tan x+\sqrt{\tan^2 x-2}|$. **1156.** $\sqrt{2}\arctan(x\sqrt{2})-\frac{1}{4(2x^2+1)}$. **1157.** $\frac{a^{\sin x}}{\ln a}$. **1158.** $\sqrt[3]{\frac{(x^3+1)^2}{2}}$. **1159.** $\frac{1}{2}\arcsin(x^2)$. **1160.** $\frac{1}{a}\tan ax-x$. **1161.** $\frac{x}{2}-\frac{\sin x}{2}$. **1162.** $\arcsin\frac{\tan x}{2}$. **1163.** $a\ln\left|\tan\left(\frac{x}{2a}+\frac{\pi}{4}\right)\right|$. **1164.** $\frac{3}{4}\sqrt[3]{(1+\ln x)^4}$. **1165.** $-2\ln|\cos\sqrt{x-1}|$. **1166.** $\frac{1}{2}\ln\left|\tan\frac{x^2}{2}\right|$. **1167.** $e^{\arctan x}+\frac{\ln^2(1+x^2)}{4}+\arctan x$. **1168.** $-\ln|\sin x+\cos x|$. **1169.** $\sqrt{2}\ln\left|\tan\frac{x}{2\sqrt{2}}\right|-2x-\sqrt{2}\cos\frac{x}{\sqrt{2}}$. **1170.** $x+\frac{1}{\sqrt{2}}\ln\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right|$. **1171.** $\ln|x|+2\arctan x$. **1172.** $e^{\sin^2 x}$. **1173.** $\frac{5}{\sqrt{3}}\arcsin\frac{x\sqrt{3}}{2}+\sqrt{4-3x^2}$. **1174.** $x-\ln(1+e^x)$. **1175.** $\frac{1}{\sqrt{a^2-b^2}}\arctan x\sqrt{\frac{a-b}{a+b}}$. **1176.** $\ln(e^x+\sqrt{e^{2x}-2})$. **1177.** $\frac{1}{a}\ln|\tan ax|$. **1178.** $-\frac{T}{2\pi}\cos\left(\frac{2\pi t}{T}+\varphi_0\right)$. **1179.** $\frac{1}{4}\ln\left|\frac{2+\ln x}{2-\ln x}\right|$. **1180.** $-\frac{\left(\arccos\frac{x}{2}\right)^2}{2}$. **1181.** $-e^{-\tan x}$. **1182.** $\frac{1}{2}\arcsin\left(\frac{\sin^2 x}{\sqrt{2}}\right)$. **1183.** $-2\cot 2x$. **1184.** $\frac{(\arcsin x)^2}{2}-\sqrt{1-x^2}$. **1185.** $\ln(\sec x+\sqrt{\sec^2 x+1})$. **1186.** $\frac{1}{4\sqrt{5}}\ln\left|\frac{\sqrt{5}+\sin 2x}{\sqrt{5}-\sin 2x}\right|$. **1187.** $\frac{1}{\sqrt{2}}\arctan\left(\frac{\tan x}{\sqrt{2}}\right)$. **Hint.** $\int\frac{dx}{1+\cos^2 x}=\int\frac{dx}{\sin^2 x+2\cos^2 x}=\int\frac{\frac{dx}{\cos^2 x}}{\tan^2 x+2}$. **1188.** $\frac{2}{3}\sqrt{[\ln(x+\sqrt{1+x^2})]^3}$. **1189.** $\frac{1}{3}\sinh(x^3+3)$. **1190.** $\frac{1}{\ln 3}3^{\tanh x}$. **1191.** a) $\frac{1}{\sqrt{2}}\arccos\frac{\sqrt{2}}{x}$ when $x>\sqrt{2}$; b) $-\ln(1+e^{-x})$;

c) $\frac{1}{80}(5x^2-3)^8$; d) $\frac{2}{3}\sqrt{(x+1)^3}-2\sqrt{x+1}$; e) $\ln(\sin x+\sqrt{1+\sin^2 x})$. 1192. $\frac{1}{4}\left[\frac{(2x+5)^{12}}{12}-\frac{5(2x+5)^{11}}{11}\right]$. 1193. $2\left(\frac{\sqrt{x^3}}{3}-\frac{x}{2}+2\sqrt{x}-2\ln|1+\sqrt{x}|\right)$. 1194. $\ln\left|\frac{\sqrt{2x+1}-1}{\sqrt{2x+1}+1}\right|$. 1195. $2\arctan\sqrt{e^x-1}$. 1196. $\ln x-\ln 2\ln|\ln x+2\ln 2|$. 1197. $\frac{(\arcsin x)^3}{3}$. 1198. $\frac{2}{3}(e^x-2)\sqrt{e^x+1}$. 1199. $\frac{2}{5}(\cos^2 x-5)\times\sqrt{\cos x}$. 1200. $\ln\left|\frac{1}{1+\sqrt{x^2+1}}\right|$. **Hint.** Put $x=\frac{1}{t}$. 1201. $-\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\arcsin x$. 1202. $-\frac{x^2}{3}\sqrt{2-x^2}-\frac{4}{3}\sqrt{2-x^2}$. 1203. $\sqrt{x^2-a^2}-a\arccos\frac{a}{x}$. 1204. $\arccos\frac{1}{x}$, if $x>0$, and $\arccos\left(-\frac{1}{x}\right)$ if $x<0$ *) **Hint.** Put $x=\frac{1}{t}$. 1205. $\sqrt{x^2+1}-\ln\left|\frac{1+\sqrt{x^2+1}}{x}\right|$. 1206. $-\frac{\sqrt{4-x^2}}{4x}$. **Note.** The substitution $x=\frac{1}{z}$ may be used in place of the trigonometric substitution. 1207. $\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\arcsin x$. 1208. $2\arcsin\sqrt{x}$. 1210. $\frac{x}{2}\sqrt{x^2-a^2}+\frac{a^2}{2}\ln|x+\sqrt{x^2-a^2}|$. 1211. $x\ln x-x$. 1212. $x\arctan x-\frac{1}{2}\ln(1+x^2)$. 1213. $x\arcsin x+\sqrt{1-x^2}$. 1214. $\sin x-x\cos x$. 1215. $\frac{x\sin 3x}{3}+\frac{\cos 3x}{9}$. 1216. $-\frac{x+1}{e^x}$. 1217. $-\frac{x\ln 2+1}{2^x\ln^2 2}$. 1218. $\frac{e^{3x}}{27}(9x^2-6x+2)$. **Solution.** In place of repeated integration by parts we can use the following method of undetermined coefficients:

$$\int x^2e^{3x}dx=(Ax^2+Bx+C)e^{3x}$$

or, after differentiation,

$$x^2e^{3x}=(Ax^2+Bx+C)3e^{3x}+(2Ax+B)e^{3x}.$$

Cancelling out e^{3x} and equating the coefficients of identical powers of x, we get:

$$1=3A;\ 0=3B+2A;\ 0=3C+B,$$

whence $A=\frac{1}{3}$; $B=-\frac{2}{9}$; $C=\frac{2}{27}$. In the general form, $\int P_n(x)e^{ax}dx=Q_n(x)e^{ax}$, where $P_n(x)$ is the given polynomial of degree n and $Q_n(x)$ is a polynomial of degree n with undetermined coefficients 1219. $-e^{-x}(x^2+5)$. **Hint.** See Problem 1218*. 1220. $-3e^{-\frac{x}{3}}(x^3+9x^2+54x+162)$. **Hint.** See

*) Henceforward, in similar cases we shall sometimes give an answer that is good for only a part of the domain of the integrand.

Problem 1218*. 1221. $-\frac{x\cos 2x}{4}+\frac{\sin 2x}{8}$. 1222. $\frac{2x^2+10x+11}{4}\sin 2x+$ $+\frac{2x+5}{4}\cos 2x$ **Hint.** It is also advisable to apply the method of undetermined coefficients in the form

$$\int P_n(x)\cos\beta x\,dx=Q_n(x)\cos\beta x+R_n(x)\sin\beta x,$$

where $P_n(x)$ is the given polynomial of degree n, and $Q_n(x)$ and $R_n(x)$ are polynomials of degree n with undetermined coefficients (see Problem 1218*).
1223. $\frac{x^3}{3}\ln x-\frac{x^3}{9}$. 1224. $x\ln^2 x-2x\ln x+2x$. 1225. $-\frac{\ln x}{2x^2}-\frac{1}{4x^2}$. 1226. $2\sqrt{x}\ln x-4\sqrt{x}$. 1227. $\frac{x^2+1}{2}\arctan x-\frac{x}{2}$. 1228. $\frac{x^2}{2}\arcsin x-\frac{1}{4}\times$ $\times\arcsin x+\frac{x}{4}\sqrt{1-x^2}$. 1229. $x\ln(x+\sqrt{1+x^2})-\sqrt{1+x^2}$. 1230. $-x\cot x+$ $+\ln|\sin x|$. 1231. $-\frac{x}{\sin x}+\ln\left|\tan\frac{x}{2}\right|$. 1232. $\frac{e^x(\sin x-\cos x)}{2}$. 1233. $\frac{3^x(\sin x+\cos x\ln 3)}{1+(\ln 3)^2}$. 1234. $\frac{e^{ax}(a\sin bx-b\cos bx)}{a^2+b^2}$. 1235. $\frac{x}{2}[\sin(\ln x)-$ $-\cos(\ln x)]$. 1236. $-\frac{e^{-x^2}}{2}(x^2+1)$. 1237. $2e^{\sqrt{x}}(\sqrt{x}-1)$. 1238. $\left(\frac{x^3}{3}-x^2+\right.$ $\left.+3x\right)\ln x-\frac{x^3}{9}+\frac{x^2}{2}-3x$. 1239. $\frac{x^2-1}{2}\ln\frac{1-x}{1+x}-x$. 1240. $-\frac{\ln^2 x}{x}-\frac{2\ln x}{x}-\frac{2}{x}$. 1241. $[\ln(\ln x)-1]\cdot\ln x$. 1242. $\frac{x^3}{3}\arctan 3x-\frac{x^2}{18}+\frac{1}{162}\ln(9x^2+1)$. 1243. $\frac{1+x^2}{2}\times$ $\times(\arctan x)^2-x\arctan x+\frac{1}{2}\ln(1+x^2)$. 1244. $x(\arcsin x)^2+2\sqrt{1-x^2}\times$ $\times\arcsin x-2x$. 1245. $-\frac{\arcsin x}{x}+\ln\left|\frac{x}{1+\sqrt{1-x^2}}\right|$. 1246. $-2\sqrt{1-x}\times$ $\times\arcsin\sqrt{x}+2\sqrt{x}$. 1247. $\frac{x\tan 2x}{2}+\frac{\ln|\cos 2x|}{4}-\frac{x^2}{2}$. 1248. $\frac{e^{-x}}{2}\times$ $\times\left(\frac{\cos 2x-2\sin 2x}{5}-1\right)$. 1249. $\frac{x}{2}+\frac{x\cos(2\ln x)+2x\sin(2\ln x)}{10}$.
1250. $-\frac{x}{2(x^2+1)}+\frac{1}{2}\arctan x$. **Solution.** Putting $u=x$ and $dv=\frac{x\,dx}{(x^2+1)^2}$, we get $du=dx$ and $v=-\frac{1}{2(x^2+1)}$. Whence $\int\frac{x^2\,dx}{(x^2+1)^2}=-\frac{x}{2(x^2+1)}+$ $+\int\frac{dx}{2(x^2+1)}=-\frac{x}{2(x^2+1)}+\frac{1}{2}\arctan x+C$. 1251. $\frac{1}{2a^2}\left(\frac{1}{a}\arctan\frac{x}{a}+\right.$ $\left.+\frac{x}{x^2+a^2}\right)$. **Hint.** Utilize the identity $1\equiv\frac{1}{a^2}[(x^2+a^2)-x^2]$. 1252. $\frac{x}{2}\times$ $\times\sqrt{a^2-x^2}+\frac{a^2}{2}\arcsin\frac{x}{a}$. **Solution.** Put $u=\sqrt{a^2-x^2}$ and $dv=dx$; whence $du=-\frac{x\,dx}{\sqrt{a^2-x^2}}$ and $v=x$; we have $\int\sqrt{a^2-x^2}\,dx=x\sqrt{a^2-x^2}-\int\frac{-x^2\,dx}{\sqrt{a^2-x^2}}=$ $=x\sqrt{a^2-x^2}-\int\frac{(a^2-x^2)-a^2}{\sqrt{a^2-x^2}}dx=x\sqrt{a^2-x^2}-\int\sqrt{a^2-x^2}\,dx+a^2\int\frac{dx}{\sqrt{a^2-x^2}}$.

Consequently, $2\int \sqrt{a^2-x^2}\,dx = x\sqrt{a^2-x^2}+a^2\arcsin\frac{x}{a}$. **1253.** $\frac{x}{2}\sqrt{A+x^2}+\frac{A}{2}\ln|x+\sqrt{A+x^2}|$. **Hint.** See Problem 1252*. **1254.** $-\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\arcsin\frac{x}{3}$. **Hint.** See Problem 1252*. **1255.** $\frac{1}{2}\arctan\frac{x+1}{2}$. **1256.** $\frac{1}{2}\times\ln\left|\frac{x}{x+2}\right|$. **1257.** $\frac{2}{\sqrt{11}}\arctan\frac{6x-1}{\sqrt{11}}$. **1258.** $\frac{1}{2}\ln(x^2-7x+13)+\frac{7}{\sqrt{3}}\times\arctan\frac{2x-7}{\sqrt{3}}$. **1259.** $\frac{3}{2}\ln(x^2-4x+5)+4\arctan(x-2)$. **1260.** $x-\frac{5}{2}\ln(x^2+3x+4)+\frac{9}{\sqrt{7}}\arctan\frac{2x+3}{\sqrt{7}}$. **1261.** $x+3\ln(x^2-6x+10)+8\arctan(x-3)$.

1262. $\frac{1}{\sqrt{2}}\arcsin\frac{4x-3}{5}$. **1263.** $\arcsin(2x-1)$. **1264.** $\ln\left|x+\frac{p}{2}+\sqrt{x^2+px+q}\right|$.

1265. $3\sqrt{x^2-4x+5}$. **1266.** $-2\sqrt{1-x-x^2}-9\arcsin\frac{2x+1}{\sqrt{5}}$.

1267. $\frac{1}{5}\sqrt{5x^2-2x+1}+\frac{1}{5\sqrt{5}}\ln\left(x\sqrt{5}-\frac{1}{\sqrt{5}}+\sqrt{5x^2-2x+1}\right)$.

1268. $\ln\left|\frac{x}{1+\sqrt{1-x^2}}\right|$. **1269.** $-\arcsin\frac{2-x}{x\sqrt{5}}$. **1270.** $\arcsin\frac{2-x}{(1-x)\sqrt{2}}\ (x>\sqrt{2})$.

1271. $-\arcsin\frac{1}{x+1}$. **1272.** $\frac{x+1}{2}\sqrt{x^2+2x+5}+2\ln(x+1+\sqrt{x^2+2x+5})$.

1273. $\frac{2x-1}{4}\sqrt{x-x^2}+\frac{1}{8}\arcsin(2x-1)$. **1274.** $\frac{2x+1}{4}\sqrt{2-x-x^2}+\frac{9}{8}\arcsin\frac{2x+1}{3}$. **1275.** $\frac{1}{4}\ln\left|\frac{x^2-3}{x^2-1}\right|$. **1276.** $-\frac{1}{\sqrt{3}}\arctan\frac{3-\sin x}{\sqrt{3}}$.

1277. $\ln\left(e^x+\frac{1}{2}+\sqrt{1+e^x+e^{2x}}\right)$ **1278.** $-\ln|\cos x+2+\sqrt{\cos^2 x+4\cos x+1}|$.

1279. $-\sqrt{1-4\ln x-\ln^2 x}-2\arcsin\frac{2+\ln x}{\sqrt{5}}$. **1280.** $\frac{1}{a-b}\ln\left|\frac{x+b}{x+a}\right|$.

1281. $x+3\ln|x-3|-3\ln|x-2|$ **1282.** $\frac{1}{12}\ln\left|\frac{(x-1)(x+3)^3}{(x+2)^4}\right|$.

1283. $\ln\left|\frac{(x-1)^4(x-4)^5}{(x+3)^7}\right|$. **1284.** $5x+\ln\left|\frac{x^{\frac{1}{2}}(x-4)^{\frac{161}{6}}}{(x-1)^{\frac{7}{3}}}\right|$. **1285.** $\frac{1}{1+x}+\ln\left|\frac{x}{x+1}\right|$.

1286. $\frac{1}{4}x+\frac{1}{16}\ln\left|\frac{x^{16}}{(2x-1)^7(2x+1)^9}\right|$ **1287.** $\frac{x^2}{2}-\frac{11}{(x-2)^2}-\frac{8}{x-2}$.

1288. $-\frac{9}{2(x-3)}-\frac{1}{2(x+1)}$. **1289.** $\frac{8}{49(x-5)}-\frac{27}{49(x+2)}+\frac{30}{343}\ln\left|\frac{x-5}{x+2}\right|$.

1290. $-\frac{1}{2(x^2-3x+2)^2}$. **1291.** $x+\ln\left|\frac{x}{\sqrt{x^2+1}}\right|$ **1292.** $x+\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\arctan x$. **1293.** $\frac{1}{52}\ln|x-3|-\frac{1}{20}\ln|x-1|+\frac{1}{65}\ln(x^2+4x+5)+\frac{7}{130}\times$

$\times \arctan(x+2)$. **1294.** $\frac{1}{6}\ln\frac{(x+1)^2}{x^2-x+1}+\frac{1}{\sqrt{3}}\arctan\frac{2x-1}{\sqrt{3}}$. **1295.** $\frac{1}{4\sqrt{2}}\times$ $\times\ln\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}+\frac{\sqrt{2}}{4}\arctan\frac{x\sqrt{2}}{1-x^2}$. **1296.** $\frac{1}{4}\ln\frac{x^2+x+1}{x^2-x+1}+\frac{1}{2\sqrt{3}}\times$ $\times\arctan\frac{x^2-1}{x\sqrt{3}}$. **1297.** $\frac{x}{2(1+x^2)}+\frac{\arctan x}{2}$. **1298.** $\frac{2x-1}{2(x^2+2x+2)}+$ $+\arctan(x+1)$. **1299.** $\ln|x+1|+\frac{x+2}{3(x^2+x+1)}+\frac{5}{3\sqrt{3}}\arctan\frac{2x+1}{\sqrt{3}}-$ $-\frac{1}{2}\ln(x^2+x+1)$. **1300.** $\frac{3x-17}{2(x^2-4x+5)}+\frac{1}{2}\ln(x^2-4x+5)+\frac{15}{2}\arctan(x-2)$.
1301. $\frac{-x^2+x}{4(x+1)(x^2+1)}+\frac{1}{2}\ln|x+1|-\frac{1}{4}\ln(x^2+1)+\frac{1}{4}\arctan x$.
1302. $-\frac{3}{8}\arctan x-\frac{x}{4(x^4-1)}+\frac{3}{16}\ln\left|\frac{x-1}{x+1}\right|$. **1303.** $\frac{15x^5+40x^3+33x}{48(1+x^2)^3}+$ $+\frac{15}{48}\arctan x$. **1304.** $x-\frac{x-1}{x^2-2x+2}+2\ln(x^2-2x+2)+3\arctan(x-1)$.
1305. $\frac{1}{21}(8\ln|x^3+8|-\ln|x^3+1|)$. **1306.** $\frac{1}{2}\ln|x^4-1|-$ $-\frac{1}{4}\ln|x^8+x^4-1|-\frac{1}{2\sqrt{5}}\ln\left|\frac{2x^4+1-\sqrt{5}}{2x^4+1+\sqrt{5}}\right|$. **1307.** $-\frac{13}{2(x-4)^2}+\frac{3}{x-4}+$ $+2\ln\left|\frac{x-4}{x-2}\right|$. **1308.** $\frac{1}{3}\left(2\ln\left|\frac{x^3+1}{x^3}\right|-\frac{1}{x^3}-\frac{1}{x^3+1}\right)$. **1309.** $\frac{1}{x-1}+$ $+\ln\left|\frac{x-2}{x-1}\right|$. **1310.** $\ln|x|-\frac{1}{7}\ln|x^7+1|$. Hint. Put $1=(x^7+1)-x^7$.
1311. $\ln|x|-\frac{1}{5}\ln|x^5+1|+\frac{1}{5(x^5+1)}$. **1312.** $\frac{1}{3}\arctan(x+1)-\frac{1}{6}\arctan\times$ $\times\frac{x+1}{2}$. **1313.** $-\frac{1}{9(x-1)^9}-\frac{1}{4(x-1)^8}-\frac{1}{7(x-1)^7}$. **1314.** $-\frac{1}{5x^5}+\frac{1}{3x^3}-\frac{1}{x}-$ $-\arctan x$. **1315.** $2\sqrt{x-1}\left[\frac{(x-1)^3}{7}+\frac{3(x-1)^2}{5}+x\right]$. **1316.** $\frac{3}{10a^2}\times$ $\times\left[2\sqrt[3]{(ax+b)^5}-5b\sqrt[3]{(ax+b)^2}\right]$ **1317.** $2\arctan\sqrt{x+1}$. **1318.** $6\sqrt[6]{x}-$ $-3\sqrt[3]{x}+2\sqrt{x}-6\ln(1+\sqrt[6]{x})$. **1319.** $\frac{6}{7}x\sqrt[6]{x}-\frac{6}{5}\sqrt[6]{x^5}-\frac{3}{2}\sqrt[3]{x^2}+$ $+2\sqrt{x}-3\sqrt[3]{x}-6\sqrt[6]{x}-3\ln|1+\sqrt[3]{x}|+6\arctan\sqrt[6]{x}$.
1320. $\ln\left|\frac{(\sqrt{x+1}-1)^2}{x+2+\sqrt{x+1}}\right|-\frac{2}{\sqrt{3}}\arctan\frac{2\sqrt{x+1}+1}{\sqrt{3}}$. **1321.** $2\sqrt{x}-2\sqrt{2}\times$ $\times\arctan\sqrt{\frac{x}{2}}$. **1322.** $-2\arctan\sqrt{1-x}$. **1323.** $\frac{\sqrt{x^2-1}}{2}(x-2)+\frac{1}{2}\ln|x+$ $+\sqrt{x^2-1}|$. **1324.** $\frac{1}{3}\ln\frac{z^2+z+1}{(z-1)^2}+\frac{2}{\sqrt{3}}\arctan\frac{2z+1}{\sqrt{3}}+\frac{2z}{z^3-1}$, where $z=\sqrt[3]{\frac{x+1}{x-1}}$. **1325.** $-\frac{\sqrt{2x+3}}{x}$. **1326.** $\frac{2x+3}{8}\sqrt{x^2-x+1}+\frac{1}{16}\ln(2x-1+$

$+2\sqrt{x^2-x+1})$. 1327. $-\frac{8+4x^2+3x^4}{15}\sqrt{1-x^2}$. 1328. $\left(\frac{5}{16}x-\frac{5}{24}x^3+\frac{1}{6}x^5\right)\times$
$\times\sqrt{1+x^2}-\frac{5}{16}\ln(x+\sqrt{1+x^2})$. 1329. $\left(\frac{1}{4x^4}+\frac{3}{8x^2}\right)\sqrt{x^2-1}-\frac{3}{8}\arcsin\frac{1}{x}$.
1330. $\frac{1}{2(x+1)^2}\sqrt{x^2+2x}-\frac{1}{2}\arcsin\frac{1}{x+1}$. 1331. $\frac{2x-1}{4}\sqrt{x^2-x+1}+\frac{19}{8}\ln\times$
$\times(2x-1+2\sqrt{x^2-x+1})$. 1332. $\frac{1}{2}\frac{1+x^2}{\sqrt{1+2x^2}}$. 1333. $\frac{1}{4}\ln\frac{\sqrt[4]{x^{-4}+1}+1}{\sqrt[4]{x^{-4}+1}-1}-$
$-\frac{1}{2}\arctan\sqrt[4]{x^{-4}+1}$. 1334. $\frac{(2x^2-1)\sqrt{1+x^2}}{3x^3}$. 1335. $\frac{1}{10}\ln\frac{(z-1)^2}{z^2+z+1}+$
$+\frac{\sqrt{3}}{5}\arctan\frac{2z+1}{\sqrt{3}}$, where $z=\sqrt[3]{1+x^5}$. 1336. $-\frac{1}{8}\frac{4+3x^3}{x(2+x^3)^{2/3}}$.
1337. $-2\sqrt[3]{(x^{-\frac{3}{4}}+1)^2}$. 1338. $\sin x-\frac{1}{3}\sin^3 x$. 1339. $-\cos x+\frac{2}{3}\cos^3 x-$
$-\frac{1}{5}\cos^5 x$. 1340. $\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}$. 1341. $\frac{1}{4}\cos^8\frac{x}{2}-\frac{1}{3}\cos^6\frac{x}{2}$. 1342. $\frac{\sin^2 x}{2}-$
$-\frac{1}{2\sin^2 x}-2\ln|\sin x|$. 1343. $\frac{3x}{8}-\frac{\sin 2x}{4}+\frac{\sin 4x}{32}$.
1344. $\frac{x}{8}-\frac{\sin 4x}{32}$. 1345. $\frac{x}{16}-\frac{\sin 4x}{64}+\frac{\sin^3 2x}{48}$. 1346. $\frac{5}{16}x+\frac{1}{12}\sin 6x+\frac{1}{64}\sin 12x+$
$+\frac{1}{144}\sin^3 6x$. 1347. $-\cot x-\frac{\cot^3 x}{3}$. 1348. $\tan x+\frac{2}{3}\tan^3 x+\frac{1}{5}\tan^5 x$.
1349. $-\frac{\cot^3 x}{3}-\frac{\cot^5 x}{5}$. 1350. $\tan x+\frac{\tan^3 x}{3}-2\cot 2x$. 1351. $\frac{1}{2}\tan^2 x+$
$+3\ln|\tan x|-\frac{3}{2\tan^2 x}-\frac{1}{4\tan^4 x}$. 1352. $\frac{1}{\cos^2\frac{x}{2}}+2\ln\left|\tan\frac{x}{2}\right|$. 1353. $\frac{\sqrt{2}}{2}\times$
$\times\left[\ln\left|\tan\frac{x}{2}\right|+\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|\right]$. 1354. $\frac{-\cos x}{4\sin^4 x}-\frac{3\cos x}{8\sin^2 x}+\frac{3}{8}\ln\left|\tan\frac{x}{2}\right|$.
1355. $\frac{\sin 4x}{16\cos^4 4x}+\frac{3\sin 4x}{32\cos^2 4x}+\frac{3}{32}\ln\left|\tan\left(2x+\frac{\pi}{4}\right)\right|$. 1356. $\frac{1}{5}\tan 5x-x$.
1357. $-\frac{\cot^2 x}{2}-\ln|\sin x|$. 1358. $-\frac{1}{3}\cot^3 x+\cot x+x$. 1359. $\frac{3}{2}\tan^2\frac{x}{3}+$
$+\tan^3\frac{x}{3}-3\tan\frac{x}{3}+3\ln\left|\cos\frac{x}{3}\right|+x$. 1360. $\frac{x^2}{4}-\frac{\sin 2x^2}{8}$. 1361. $-\frac{\cot^3 x}{3}$.
1362. $-\frac{3}{4}\sqrt[3]{\cos^4 x}+\frac{3}{5}\sqrt[3]{\cos^{10} x}-\frac{3}{16}\sqrt[3]{\cos^{16} x}$. 1363. $2\sqrt{\tan x}$. 1364. $\frac{1}{2\sqrt{2}}\times$
$\times\ln\frac{z^2+z\sqrt{2}+1}{z^2-z\sqrt{2}+1}-\frac{1}{\sqrt{2}}\arctan\frac{z\sqrt{2}}{z^2-1}$, where $z=\sqrt{\tan x}$. 1365. $-\frac{\cos 8x}{16}+$
$+\frac{\cos 2x}{4}$. 1366. $-\frac{\sin 25x}{50}+\frac{\sin 5x}{10}$. 1367. $\frac{3}{5}\sin\frac{5x}{6}+3\sin\frac{x}{6}$. 1368. $\frac{3}{2}\cos\frac{x}{3}-$
$-\frac{1}{2}\cos x$. 1369. $\frac{\sin 2ax}{4a}+\frac{x\cos 2b}{2}$. 1370. $\frac{t\cos\varphi}{2}-\frac{\sin(2\omega t+\varphi)}{4\omega}$. 1371. $\frac{\sin x}{2}+$

$+\frac{\sin 5x}{20}+\frac{\sin 7x}{28}$. **1372.** $\frac{1}{24}\cos 6x-\frac{1}{16}\cos 4x-\frac{1}{8}\cos 2x$. **1373.** $\frac{1}{4}\ln\left|\frac{\tan\frac{x}{2}-2}{\tan\frac{x}{2}+2}\right|$.

1374. $\frac{1}{\sqrt{2}}\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)\right|$. **1375.** $x-\tan\frac{x}{2}$. **1376.** $-x+\tan x+\sec x$.

1377. $\ln\left|\frac{\tan\frac{x}{2}-5}{\tan\frac{x}{2}-3}\right|$. **1378.** $\arctan\left(1+\tan\frac{x}{2}\right)$. **1379.** $\frac{12}{13}x-\frac{5}{13}\ln|2\sin x+$ $+3\cos x|$ **Solution.** We put $3\sin x+2\cos x\equiv\alpha(2\sin x+3\cos x)+$ $+\beta(2\sin x+3\cos x)'$ Whence $2\alpha-3\beta=3$, $3\alpha+2\beta=2$ and, consequently, $\alpha=\frac{12}{13}$, $\beta=-\frac{5}{13}$. We have $\int\frac{3\sin x+2\cos x}{2\sin x+3\cos x}dx=\frac{12}{13}\int dx-\frac{5}{13}\times$ $\times\int\frac{(2\sin x+3\cos x)'}{2\sin x+3\cos x}dx=\frac{12}{13}x-\frac{5}{13}\ln|2\sin x+3\cos x|$. **1380.** $-\ln|\cos x-\sin x|$.

1381. $\frac{1}{2}\arctan\left(\frac{\tan x}{2}\right)$ **Hint.** Divide the numerator and denominator of the fraction by $\cos^2 x$ **1382.** $\frac{1}{\sqrt{15}}\arctan\left(\frac{\sqrt{3}\tan x}{\sqrt{5}}\right)$. **Hint.** See Problem 1381.

1383. $\frac{1}{\sqrt{3}}\ln\left|\frac{2\tan x+3-\sqrt{13}}{2\tan x+3+\sqrt{13}}\right|$. **Hint.** See Problem 1381. **1384.** $\frac{1}{5}\ln\times$ $\times\left|\frac{\tan x-5}{\tan x}\right|$ **Hint.** See Problem 1381. **1385.** $-\frac{1}{2(1-\cos x)^2}$. **1386.** $\ln(1+\sin^2 x)$.

1387. $\frac{1}{2\sqrt{2}}\ln\frac{\sqrt{2}+\sin 2x}{\sqrt{2}-\sin 2x}$. **1388.** $\frac{1}{4}\ln\frac{5-\sin x}{1-\sin x}$. **1389.** $\frac{2}{\sqrt{3}}\arctan\times$ $\times\frac{2\tan\frac{x}{2}-1}{\sqrt{3}}-\frac{1}{\sqrt{2}}\arctan\frac{3\tan\frac{x}{2}-1}{2\sqrt{2}}$. **Hint.** Use the identity $\frac{1}{(2-\sin x)(3-\sin x)}\equiv\frac{1}{2-\sin x}-\frac{1}{3-\sin x}$. **1390.** $-x+2\ln\left|\frac{\tan\frac{x}{2}}{\tan\frac{x}{2}+1}\right|$. **Hint.** Use the identity $\frac{1-\sin x+\cos x}{1+\sin x-\cos x}\equiv-1+\frac{2}{1+\sin x-\cos x}$. **1391.** $\frac{\cosh^3 x}{3}-\cosh x$.

1392. $\frac{3x}{8}+\frac{\sinh 2x}{4}+\frac{\sinh 4x}{32}$. **1393.** $\frac{\sinh^4 x}{4}$. **1394.** $-\frac{x}{8}+\frac{\sinh 4x}{32}$.

1395. $\ln\left|\tanh\frac{x}{2}\right|+\frac{1}{\cosh x}$. **1396.** $-2\coth 2x$. **1397.** $\ln(\cosh x)-\frac{\tanh^2 x}{2}$.

1398. $x-\coth x-\frac{\coth^3 x}{3}$. **1399.** $\arctan(\tanh x)$. **1400.** $\frac{2}{\sqrt{5}}\arctan\left(\frac{3\tanh\frac{x}{2}+2}{\sqrt{5}}\right)$ $\left[\text{or } \frac{2}{\sqrt{5}}\arctan(e^x\sqrt{5})\right]$. **1401.** $\frac{\sinh^2 x}{2}-\frac{\sinh 2x}{4}-\frac{x}{2}$. **Hint.** Use the identity $\frac{-1}{\sinh x-\cosh x}\equiv(\sinh x+\cosh x)$. **1402.** $\frac{1}{\sqrt{2}}\ln(\sqrt{2}\cosh x+\sqrt{\cosh 2x})$.

1403. $\frac{x+1}{2}\sqrt{3-2x-x^2}+2\arcsin\frac{x+1}{2}$. 1404. $\frac{x}{2}\sqrt{2+x^2}+\ln(x+\sqrt{2+x^2})$.
1405. $\frac{x}{2}\sqrt{9+x^2}-\frac{9}{2}\ln(x+\sqrt{9+x^2})$. 1406. $\frac{x-1}{2}\sqrt{x^2-2x+2}+$ $+\frac{1}{2}\ln(x-1+\sqrt{x^2-2x+2})$ 1407. $\frac{x}{2}\sqrt{x^2-4}-2\ln|x+\sqrt{x^2-4}|$.
1408. $\frac{2x+1}{4}\sqrt{x^2+x}-\frac{1}{8}\ln|2x+1+2\sqrt{x^2+x}|$. 1409. $\frac{x-3}{2}\sqrt{x^2-6x-7}-$ $-8\ln|x-3+\sqrt{x^2-6x-7}|$. 1410. $\frac{1}{64}(2x+1)(8x^2+8x+17)\sqrt{x^2+x+1}+$ $+\frac{27}{128}\ln(2x+1+2\sqrt{x^2+x+1})$. 1411. $2\sqrt{\frac{x-2}{x-1}}$ 1412. $\frac{x-1}{4\sqrt{x^2-2x+5}}$.
1413. $\frac{1}{\sqrt{2}}\arctan\frac{x\sqrt{2}}{\sqrt{1-x^2}}$ 1414. $\frac{1}{2\sqrt{2}}\ln\left|\frac{\sqrt{1+x^2}+x\sqrt{2}}{\sqrt{1+x^2}-x\sqrt{2}}\right|$ 1415. $\frac{e^{2x}}{2}\times$ $\times\left(x^4-2x^3+5x^2-5x+\frac{7}{2}\right)$ 1416. $\frac{1}{6}\left(x^3+\frac{x^2}{2}\sin 6x+\frac{x}{6}\cos 6x-\frac{1}{36}\sin 6x\right)$.
1417. $-\frac{x\cos 3x}{6}+\frac{\sin 3x}{18}+\frac{x\cos x}{2}-\frac{\sin x}{2}$ 1418. $\frac{e^{2x}}{8}(2-\sin 2x-\cos 2x)$.
1419. $\frac{e^x}{2}\left(\frac{2\sin 2x+\cos 2x}{5}-\frac{4\sin 4x+\cos 4x}{17}\right)$. 1420. $\frac{e^x}{2}[x(\sin x+\cos x)-\sin x]$.
1421. $-\frac{x}{2}+\frac{1}{3}\ln|e^x-1|+\frac{1}{6}\ln(e^x+2)$ 1422. $x-\ln(2+e^x+2\sqrt{e^{2x}+x+1})$.
1423. $\frac{1}{3}\left[x^3\ln\frac{1+x}{1-x}+\ln(1-x^2)+x^2\right]$ 1424. $x\ln^2(x+\sqrt{1+x^2})-2\sqrt{1+x^2}\times$ $\times\ln(x+\sqrt{1+x^2})+2x$. 1425. $\left(\frac{x^2}{2}-\frac{9}{100}\right)\arccos(5x-2)-\frac{5x+6}{100}\times$ $\times\sqrt{20x-25x^2-3}$. 1426. $\frac{\sin x\cosh x-\cos x\sinh x}{2}$. 1427. $I_n=\frac{1}{2(n-1)a^2}\times$ $\times\left[\frac{x}{(x^2+a^2)^{n-1}}+(2n-3)I_{n-1}\right]$; $I_2=\frac{1}{2a^2}\left(\frac{x}{x^2+a^2}+\frac{1}{a}\arctan\frac{x}{a}\right)$; $I_3=\frac{1}{4a^2}\times$ $\times\left[\frac{x(3x^2+5a^2)}{2a^2(x^2+a^2)^2}+\frac{3}{2a^3}\arctan\frac{x}{a}\right]$. 1428. $I_n=-\frac{\cos x\sin^{n-1}x}{n}+\frac{n-1}{n}I_{n-2}$; $I_4=\frac{3x}{8}-\frac{\cos x\sin^3 x}{4}-\frac{3\sin 2x}{16}$; $I_5=-\frac{\cos x\sin^4 x}{5}-\frac{4}{15}\cos x\sin^2 x-\frac{8}{15}\cos x$.
1429. $I_n=\frac{\sin x}{(n-1)\cos^{n-1}x}+\frac{n-2}{n-1}I_{n-2}$; $I_3=\frac{\sin x}{2\cos^2 x}+\frac{1}{2}\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$; $I_4=\frac{\sin x}{3\cos^3 x}+\frac{2}{3}\tan x$. 1430. $I_n=-x^n e^{-x}+nI_{n-1}$; $I_{10}=-e^{-x}(x^{10}+10x^9+$ $+10\cdot 9x^8+\ldots+10\cdot 9\cdot 8\ldots 2x+10\cdot 9\ldots 1)$. 1431. $\frac{1}{\sqrt{14}}\arctan\frac{\sqrt{2}(x-1)}{\sqrt{7}}$.
1432. $\ln\sqrt{x^2-2x+2}-4\arctan(x-1)$. 1433. $\frac{(x-1)^2}{2}+\frac{1}{4}\ln\left(x^2+x+\frac{1}{2}\right)+$ $+\frac{1}{2}\arctan(2x+1)$. 1434. $\frac{1}{5}\ln\sqrt{\frac{x^2}{x^2+5}}$. 1435. $2\ln\left|\frac{x+3}{x+2}\right|-\frac{1}{x+2}-\frac{1}{x+3}$.
1436. $\frac{1}{2}\left(\ln\left|\frac{x+1}{\sqrt{x^2+1}}\right|-\frac{1}{x+1}\right)$. 1437. $\frac{1}{4}\left(\frac{x}{x^2+2}+\frac{1}{\sqrt{2}}\arctan\frac{x}{\sqrt{2}}\right)$.

1438. $\frac{1}{4}\left(\frac{2x}{1-x^2}+\ln\left|\frac{x+1}{x-1}\right|\right)$. 1439. $\frac{1}{6}\frac{x-2}{(x^2-x+1)^2}+\frac{1}{6}\frac{2x-1}{x^2-x+1}+\frac{2}{3\sqrt{3}}\arctan\frac{2x-1}{\sqrt{3}}$. 1440. $\frac{x(3+2\sqrt{x})}{1-2\sqrt{x}}$. 1441. $-\frac{1}{x}-\frac{4}{3x\sqrt{x}}-\frac{1}{2x^2}$. 1442. $\ln\left(x+\frac{1}{2}+\sqrt{x^2+x+1}\right)$. 1443. $\sqrt{2x}-\frac{3}{5}\sqrt[6]{(2x)^5}$. 1444. $-\frac{3}{\sqrt[3]{x+1}}$.

1445. $\frac{2x-1}{\sqrt{4x^2-2x+1}}$. 1446. $-2(\sqrt[4]{5-x}-1)^2-4\ln(1+\sqrt[4]{5-x})$.

1477. $\ln|x+\sqrt{x^2-1}|-\frac{x}{\sqrt{x^2-1}}$. 1448. $-\frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}$. 1449. $\frac{1}{2}\times\arcsin\frac{x^2+1}{\sqrt{2}}$. 1450. $\frac{x-1}{\sqrt{x^2+1}}$. 1451. $\frac{1}{8}\ln\left|\frac{\sqrt{4-x^2}-2}{x}\right|-\frac{1}{8\sqrt{3}}\times\arcsin\frac{2(x+1)}{x+4}$. Hint. $\frac{1}{x^2+4x}=\frac{1}{4}\left(\frac{1}{x}-\frac{1}{x+4}\right)$. 1452. $\frac{x}{2}\sqrt{x^2-9}-\frac{9}{2}\ln|x+\sqrt{x^2-9}|$. 1453. $\frac{1}{16}(8x-1)\sqrt{x-4x^2}+\frac{1}{64}\arcsin(8x-1)$.

1454. $\ln\left|\frac{x}{2x+1+2\sqrt{x^2+x+1}}\right|$. 1455. $\frac{(x^2+2x+2)\sqrt{x^2+2x+2}}{3}-\frac{(x+1)}{2}\sqrt{x^2+2x+2}-\frac{1}{2}\ln(x+1+\sqrt{x^2+2x+2})$. 1456. $\frac{\sqrt{x^2-1}}{x}-\frac{\sqrt{(x^2-1)^3}}{3x^3}$. 1457. $\frac{1}{3}\ln\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right|$. 1458. $-\frac{1}{3}\ln|z-1|+\frac{1}{6}\ln(z^2+z+1)-\frac{1}{\sqrt{3}}\arctan\frac{2z+1}{\sqrt{3}}$, where $z=\frac{\sqrt[3]{1+x^3}}{x}$. 1459. $\frac{5}{2}\times\ln(x^2+\sqrt{1+x^4})$. 1460. $\frac{3x}{8}+\frac{\sin 2x}{4}+\frac{\sin 4x}{32}$. 1461. $\ln|\tan x|-\cot^2 x-\frac{1}{4}\cot^4 x$. 1462. $-\cot x-\frac{2\sqrt{(\cot x)^3}}{3}$. 1463. $\frac{5}{12}(\cos^2 x-6)\sqrt[5]{\cos^2 x}$.

1464. $-\frac{\cos 5x}{20\sin^4 5x}-\frac{3\cos 5x}{40\sin^2 5x}+\frac{3}{40}\ln\left|\tan\frac{5x}{2}\right|$. 1465. $\frac{\tan^3 x}{3}+\frac{\tan^5 x}{5}$.

1466. $\frac{1}{4}\sin 2x$. 1467. $\tan^2\left(\frac{x}{2}+\frac{\pi}{4}\right)+2\ln\left|\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$. 1468. $-\frac{1}{\sqrt{3}}\times\arctan\frac{4\tan\frac{x}{2}-1}{\sqrt{3}}$. 1469. $\frac{1}{\sqrt{10}}\arctan\left(\frac{2\tan x}{\sqrt{10}}\right)$. 1470. $\arctan(2\tan x+1)$.

1471. $\frac{1}{2}\ln|\tan x+\sec x|-\frac{1}{2}\operatorname{cosec} x$. 1472. $\frac{2}{\sqrt{3}}\times\arctan\left(\frac{\tan\frac{x}{2}}{\sqrt{3}}\right)-\frac{1}{\sqrt{2}}\times\arctan\left(\frac{\tan\frac{x}{2}}{\sqrt{2}}\right)$. 1473. $\ln|\tan x+2+\sqrt{\tan^2 x+4\tan x+1}|$. 1474. $\frac{1}{a}\times\ln(\sin ax+\sqrt{a^2+\sin^2 ax})$. 1475. $\frac{1}{3}x\tan 3x+\frac{1}{9}\ln|\cos 3x|$. 1476. $\frac{x^2}{4}-\frac{x\sin 2x}{4}-\frac{\cos 2x}{8}$. 1477. $\frac{e^{2x}}{4}(2x-1)$. 1478 $\frac{1}{3}e^{x^3}$. 1479. $\frac{x^3}{3}\cdot\ln\sqrt{1-x}-$

$-\frac{1}{6}\ln|x-1|-\frac{x^3}{18}-\frac{x^2}{12}-\frac{x}{6}$. **1480.** $\sqrt{1+x^2}\,\mathrm{arc\,tan}\,x-\ln(x+\sqrt{1+x^2})$. **1481.** $\frac{1}{3}\sin\frac{3x}{2}-\frac{1}{10}\sin\frac{5x}{2}-\frac{1}{2}\sin\frac{x}{2}$. **1482.** $-\frac{1}{1+\tan x}$. **1483.** $\ln|1+\cot x|-\cot x$. **1481.** $\frac{\sinh^2 x}{2}$. **1485.** $-2\cosh\sqrt{1-x}$. **1486.** $\frac{1}{5}\ln\cosh 2x$. **1487.** $-x\coth x+\ln|\sinh x|$. **1488.** $\frac{1}{2e^x}-\frac{x}{4}+\frac{1}{4}\ln|e^x-2|$. **1489.** $\frac{1}{2}\,\mathrm{arc\,tan}\,\frac{e^x-3}{2}$. **1490.** $\frac{4}{7}\sqrt[4]{(e^x+1)^7}-\frac{4}{3}\sqrt[4]{(e^x+1)^3}$. **1491.** $\frac{1}{\ln 4}\ln\frac{1+2^x}{1-2^x}$. **1492.** $-\frac{10^{-2x}}{2\ln 10}\times\left(x^2-1+\frac{x}{\ln 10}+\frac{1}{2\ln^2 10}\right)$. **1493.** $2\sqrt{e^x+1}+\ln\frac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1}$. **1494.** $\ln\left|\frac{x}{\sqrt{1+x^2}}\right|-\frac{\mathrm{arc\,tan}\,x}{x}$. **1495.** $\frac{1}{4}\left(x^4\arcsin\frac{1}{x}+\frac{x^2+2}{3}\sqrt{x^2-1}\right)$. **1496.** $\frac{x}{2}(\cos\ln x+\sin\ln x)$. **1497.** $\frac{1}{5}\left(-x^2\cos 5x+\frac{2}{5}x\sin 5x+3x\cos 5x+\frac{2}{25}\cos 5x-\frac{3}{5}\sin 5x\right)$. **1498.** $\frac{1}{2}\left[(x^2-2)\,\mathrm{arc\,tan}\,(2x+3)+\frac{3}{4}\ln(2x^2+6x+5)-\frac{x}{2}\right]$. **1499.** $\frac{1}{2}\sqrt{x-x^2}+\left(x-\frac{1}{2}\right)\arcsin\sqrt{x}$. **1500.** $\frac{x|x|}{2}$.

Chapter V

1501. $b-a$. **1502.** $v_0T-g\frac{T^2}{2}$. **1503.** 3. **1504.** $\frac{2^{10}-1}{\ln 2}$. **1505.** 156. **Hint.** Divide the interval from $x=1$ to $x=5$ on the x-axis into subintervals so that the abscissas of the points of division should form a geometric progression: $x_0=1$, $x_1=x_0q$, $x_2=x_0q^2$, ..., $x_n=x_0q^n$. **1506.** $\ln\frac{b}{a}$. **Hint.** See Problem 1505. **1507.** $1-\cos x$. **Hint.** Utilize the formula $\sin\alpha+\sin 2\alpha+\ldots+\sin n\alpha=\frac{1}{2\sin\frac{\alpha}{2}}\left[\cos\frac{\alpha}{2}-\cos\left(n+\frac{1}{2}\right)\alpha\right]$. **1508.** 1) $\frac{dI}{da}=-\frac{1}{\ln a}$; 2) $\frac{dI}{db}=\frac{1}{\ln b}$. **1509.** $\ln x$. **1510.** $-\sqrt{1+x^4}$. **1511.** $2xe^{-x^4}-e^{-x^2}$. **1512.** $\frac{\cos x}{2\sqrt{x}}+\frac{1}{x^2}\cos\frac{1}{x^2}$. **1513.** $x=n\pi$ $(n=1, 2, 3, \ldots)$. **1514.** $\ln 2$. **1515.** $-\frac{3}{8}$. **1516.** $e^x-e^{-x}=2\sinh x$. **1517.** $\sin x$. **1518.** $\frac{1}{2}$. **Solution.** The sum $s_n=\frac{1}{n^2}+\frac{2}{n^2}+\ldots+\frac{n-1}{n^2}=\frac{1}{n}\left(\frac{1}{n}+\frac{2}{n}+\ldots+\frac{n-1}{n}\right)$ may be regarded as the integral sum of the function $f(x)=x$ on the interval $[0,1]$. Therefore, $\lim_{n\to\infty}s_n=\int_0^1 x\,dx=\frac{1}{2}$. **1519.** $\ln 2$. **Solution.** The sum $s_n=\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{n+n}=\frac{1}{n}\left(\frac{1}{1+\frac{1}{n}}+\frac{1}{1+\frac{2}{n}}+\ldots+\frac{1}{1+\frac{n}{n}}\right)$ may be regarded as the integral sum of the function $f(x)=\frac{1}{1+x}$ on the interval [0,1] where the division points have the form $x_k=1+\frac{k}{n}$ $(k=1, 2, \ldots, n)$. Therefore, $\lim\limits_{n\to\infty} s_n=\int\limits_0^1 \frac{dx}{1+x}=\ln 2$.

1520. $\frac{1}{p+1}$. 1521. $\frac{7}{3}$. 1522. $\frac{100}{3}=33\frac{1}{3}$. 1523. $\frac{7}{4}$. 1524. $\frac{16}{3}$. 1525. $-\frac{2}{3}$. 1526. $\frac{1}{2}\ln\frac{2}{3}$. 1527. $\ln\frac{9}{8}$. 1528. $35\frac{1}{15}-32\ln 3$. 1529. $\arctan 3-\arctan 2=$ $=\arctan\frac{1}{7}$. 1530. $\ln\frac{4}{3}$. 1531. $\frac{\pi}{16}$. 1532. $1-\frac{1}{\sqrt{3}}$. 1533. $\frac{\pi}{4}$. 1534. $\frac{\pi}{2}$. 1535. $\frac{1}{3}\ln\frac{1+\sqrt{5}}{2}$. 1536. $\frac{\pi}{8}+\frac{1}{4}$. 1537. $\frac{2}{3}$. 1538. $\ln 2$. 1539. $1-\cos 1$. 1540. 0. 1541. $\frac{8}{9\sqrt{3}}+\frac{\pi}{6}$. 1542. $\arctan e-\frac{\pi}{4}$. 1543. $\sinh 1=\frac{1}{2}\left(e-\frac{1}{e}\right)$. 1544. $\tanh(\ln 3)-\tanh(\ln 2)=\frac{1}{5}$. 1545. $-\frac{\pi}{2}+\frac{1}{4}\sinh 2\pi$. 1546. 2. 1547. Diverges. 1548. $\frac{1}{1-p}$, if $p<1$; diverges, if $p\geqslant 1$. 1549. Diverges. 1550. $\frac{\pi}{2}$. 1551. Diverges. 1552. 1. 1553. $\frac{1}{p-1}$, if $p>1$; diverges, if $p\leqslant 1$. 1554. π. 1555. $\frac{\pi}{\sqrt{5}}$. 1556. Diverges. 1557. Diverges. 1558. $\frac{1}{\ln 2}$ 1559. Diverges. 1560. $\frac{1}{\ln a}$. 1561. Diverges. 1562. $\frac{1}{k}$ 1563. $\frac{\pi^2}{8}$. 1564. $\frac{1}{3}+\frac{1}{4}\ln 3$ 1565. $\frac{2\pi}{3\sqrt{3}}$. 1566. Diverges 1567. Converges 1568. Diverges 1569. Converges. 1570. Converges. 1571. Converges. 1572. Diverges 1573. Converges. 1574. Hint. $B(p, q)=$ $=\int\limits_0^{1/2} f(x)\,dx+\int\limits_{1/2}^1 f(x)\,dx$, where $f(x)=x^{p-1}(1-x)^{q-1}$; since $\lim\limits_{x\to 0} f(x)\,x^{1-p}=1$ and $\lim\limits_{x\to 1}(1-x)^{1-q}f(x)=1$, both integrals converge when $1-p<1$ and $1-q<1$, that is, when $p>0$ and $q>0$. 1575. Hint. $\Gamma(p)=\int\limits_0^1 f(x)\,dx+\int\limits_1^\infty f(x)\,dx$, where $f(x)=x^{p-1}e^{-x}$. The first integral converges when $p>0$, the second when p is arbitrary. 1576. No. 1577. $2\sqrt{2}\int\limits_1^2 \sqrt{t}\,dt$. 1578. $\int\limits_{\pi/6}^{\pi/2}\frac{dt}{\sqrt{1+\sin^2 t}}$. 1579. $\int\limits_{\ln 2}^{\ln 3} dt$.

1580. $\int\limits_0^\infty \frac{f(\arctan t)}{1+t^2}\,dt$. 1581. $x=(b-a)\,t+a$. 1582. $4-2\ln 3$. 1583. $8-\frac{9}{2\sqrt{3}}\pi$. 1584. $2-\frac{\pi}{2}$. 1585. $\frac{\pi}{\sqrt{5}}$ 1586. $\frac{\pi}{2\sqrt{1+a^2}}$. 1587. $1-\frac{\pi}{4}$. 1588. $\sqrt{3}-\frac{\pi}{3}$. 1589. $4-\pi$. 1590. $\frac{1}{5}\ln 112$. 1591. $\ln\frac{7+2\sqrt{7}}{9}$. 1592. $\frac{1}{2}+\frac{\pi}{4}$. 1593. $\frac{\pi a^2}{8}$

1594. $\frac{\pi}{2}$. 1599. $\frac{\pi}{2}-1$. 1600. 1. 1601. $\frac{e^2+3}{8}$. 1602. $\frac{1}{2}(e^{\pi}+1)$. 1603. 1. 1604. $\frac{a}{a^2+b^2}$. 1605. $\frac{b}{a^2+b^2}$. 1606. **Solution.** $\Gamma(p+1)=\int_0^{\infty} x^p e^{-x}\,dx$. Applying the formula of integration by parts, we put $x^p=u$, $e^{-x}dx=dv$. Whence

$$du=px^{p-1}\,dx, \quad v=-e^{-x}$$

and

$$\Gamma(p+1)=[-x^pe^{-x}]_0^{\infty}+p\int_0^{\infty} x^{p-1}e^{-x}\,dx=p\Gamma(p) \qquad (*)$$

If p is a natural number, then, applying formula (*) p times and taking into account that

$$\Gamma(1)=\int_0^{\infty} e^{-x}\,dx=1,$$

we get:

$$\Gamma(p+1)=p!$$

1607. $I_{2k}=\frac{1\cdot3\cdot5\,\ldots\,(2k-1)}{2\cdot4\cdot6\,\ldots\,2k}\frac{\pi}{2}$, if $n=2k$ is an even number; $I_{2k+1}=\frac{2\cdot4\cdot6\,\ldots\,2k}{1\cdot3\cdot5\,\ldots\,(2k+1)}$, if $n=2k+1$ is an odd number

$$I_9=\frac{128}{315}; \quad I_{10}=\frac{63\pi}{512}.$$

1608. $\frac{(p-1)!\,(q-1)!}{(p+q-1)!}$. 1609. $\frac{1}{2}\,\mathrm{B}\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$. **Hint.** Put $\sin^2 x=t$. 1610. a) Plus; b) minus; c) plus **Hint.** Sketch the graph of the integrand for values of the argument on the interval of integration 1611. a) First; b) second; c) first. 1612. $\frac{1}{3}$ 1613. a. 1614. $\frac{1}{2}$. 1615. $\frac{3}{8}$. 1616. $2\arcsin\frac{1}{3}$. 1617. $2<I<\sqrt{5}$. 1618. $\frac{2}{9}<I<\frac{2}{7}$. 1619. $\frac{2}{13}\pi<I<\frac{2}{7}\pi$. 1620. $0<I<\frac{\pi^2}{32}$. **Hint.** The integrand increases monotonically. 1621. $\frac{1}{2}<I<\frac{\sqrt{2}}{2}$. 1623. $s=\frac{32}{3}$. 1624. 1. 1625. $\frac{1}{2}$ **Hint.** Take account of the sign of the function. 1626. $4\frac{1}{4}$. 1627. 2. 1628. $\ln 2$. 1629. $m^2\ln 3$. 1630. πa^2. 1631. 12. 1632. $\frac{4}{3}p^2$. 1633. $4\frac{1}{2}$. 1634. $10\frac{2}{3}$. 1635. 4. 1636. $\frac{32}{3}$. 1637. $\frac{\pi}{2}-\frac{1}{3}$. 1638. $e+\frac{1}{e}-2=2(\cosh 1-1)$. 1639. $ab\,[2\sqrt{3}-\ln(2+\sqrt{3})]$. 1640. $\frac{3}{8}\pi a^2$. **Hint.** See Appendix VI, Fig. 27. 1641. $2a^2e^{-1}$. 1642. $\frac{4}{3}a^2$. 1643. 15π. 1644. $\frac{9}{2}\ln 3$. 1645. 1. 1646. $3\pi a^2$. **Hint.** See Appendix VI, Fig. 23. 1647. $a^2\left(2+\frac{\pi}{2}\right)$. **Hint.** See Appendix VI, Fig. 24. 1648. $2\pi+\frac{4}{3}$ and $6\pi-\frac{4}{3}$. 1649. $\frac{16}{3}\pi-\frac{4\sqrt{3}}{3}$ and $\frac{32}{3}\pi+\frac{4\sqrt{3}}{3}$. 1650. $\frac{3}{8}\pi ab$.

1651. $3\pi a^2$. **1652.** $\pi(b^2+2ab)$. **1653.** $6\pi a^2$. **1654.** $\frac{3}{2}a^2$. **Hint.** For the loop, the parameter t varies within the limits $0 \leqslant t \leqslant +\infty$ See Appendix VI, Fig. 22. **1655.** $\frac{3}{2}\pi a^2$. **Hint.** See Appendix VI, Fig. 28. **1656.** $8\pi^3 a^2$. **Hint.** See Appendix VI, Fig. 30. **1657.** $\frac{\pi a^2}{8}$. **1658.** a^2. **1659.** $\frac{\pi a^2}{4}$. **Hint.** See Appendix VI, Fig. 33. **1660.** $\frac{9}{2}\pi$. **1661.** $\frac{14-8\sqrt{2}}{3}a^2$. **1662.** $\frac{\pi p^2}{(1-\varepsilon^2)^{3/2}}$. **1663.** $a^2\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)$. **1664.** $\pi\sqrt{2}$. **Hint.** Pass to polar coordinates. **1665.** $\frac{8}{27}(10\sqrt{10}-1)$. **1666.** $\sqrt{h^2-a^2}$. **Hint.** Utilize the formula $\cosh^2\alpha-\sinh^2\alpha=1$. **1667.** $\sqrt{2}+\ln(1+\sqrt{2})$. **1668.** $\sqrt{1+e^2}-\sqrt{2}+\ln\frac{(\sqrt{1+e^2}-1)(\sqrt{2}+1)}{e}$. **1669.** $1+\frac{1}{2}\ln\frac{3}{2}$. **1670.** $\ln(e+\sqrt{e^2-1})$. **1671.** $\ln(2+\sqrt{3})$ **1672.** $\frac{1}{4}(e^2+1)$. **1673.** $a\ln\frac{a}{b}$. **1674.** $2a\sqrt{3}$. **1675.** $\ln\frac{e^{2b}-1}{e^{2a}-1}+a-b=\ln\frac{\sinh b}{\sinh a}$. **1676.** $\frac{1}{2}aT^2$. **Hint.** See Appendix VI, Fig. 29. **1677.** $\frac{4(a^3-b^3)}{ab}$. **1678.** $16a$. **1679.** $\pi a\sqrt{1+4\pi^2}+\frac{a}{2}\ln(2\pi+\sqrt{1+4\pi^2})$. **1680.** $8a$. **1681.** $2a[\sqrt{2}+\ln(\sqrt{2}+1)]$. **1682.** $\frac{\sqrt{5}}{2}+\ln\frac{3+\sqrt{5}}{2}$. **1683.** $\frac{a\sqrt{1+m^2}}{m}$. **1684.** $\frac{1}{2}[4+\ln 3]$. **1685.** $\frac{\pi a^5}{30}$. **1686.** $\frac{4}{3}\pi ab^2$. **1687.** $\frac{a^3\pi}{4}(e^2+4-e^{-2})$. **1688.** $\frac{3}{8}\pi^2$. **1689.** $v_x=\frac{\pi}{4}$. **1690.** $v_y=\frac{4}{7}\pi$. **1691.** $v_x=\frac{\pi}{2}$; $v_y=2\pi$. **1692.** $\frac{16\pi a^3}{5}$. **1693.** $\frac{32}{15}\pi a^3$. **1694.** $\frac{4}{3}\pi p^3$. **1695.** $\frac{3}{10}\pi$. **1696.** $\frac{\pi a^3}{2}(15-16\ln 2)$. **1697.** $2\pi^2 a^3$. **1698.** $\frac{\pi R^2 H}{2}$. **1699.** $\frac{16}{15}\pi h^2 a$. **1701.** a) $5\pi^2 a^3$; b) $6\pi^3 a^3$; c) $\frac{\pi a^3}{6}(9\pi^2-16)$. **1702.** $\frac{32}{105}\pi a^3$. **1703.** $\frac{8}{3}\pi a^3$. **1704.** $\frac{4}{21}\pi a^3$. **1705.** $\frac{h}{3}\left(AB+\frac{Ab+aB}{2}+ab\right)$. **1706.** $\frac{\pi abh}{3}$. **1707.** $\frac{128}{105}a^3$. **1708.** $\frac{8}{3}\pi a^2 b$. **1709.** $\frac{1}{2}\pi a^2 h$. **1710.** $\frac{16}{3}a^3$. **1711.** $\pi a^2\sqrt{pq}$. **1712.** $\pi abh\left(1+\frac{h^2}{3c^2}\right)$. **1713.** $\frac{4}{3}\pi abc$. **1714.** $\frac{8\pi}{3}[\sqrt{17^3}-1]$; $\frac{16}{3}\pi a^2[5\sqrt{5}-8]$. **1715.** $2\pi[\sqrt{2}+\ln(\sqrt{2}+1)]$. **1716.** $\pi(\sqrt{5}-\sqrt{2})+\pi\ln\frac{2(\sqrt{2}+1)}{\sqrt{5}+1}$. **1717.** $\pi[\sqrt{2}+\ln(1+\sqrt{2})]$. **1718.** $\frac{\pi a^2}{4}(e^2+e^{-2}+4)=\frac{\pi a^2}{2}(2+\sinh 2)$. **1719.** $\frac{12}{5}\pi a^2$. **1720.** $\frac{\pi}{3}(e-1)(e^2+e+4)$. **1721.** $4\pi^2 ab$ **Hint.** Here, $y=b\pm\sqrt{a^2-x^2}$. Taking the plus sign, we get the external surface of a torus; taking the minus sign, we get the internal surface of a torus. **1722.** 1) $2\pi b^2+\frac{2\pi ab}{\varepsilon}\arcsin\varepsilon$; 2) $2\pi a^2+\frac{\pi b^2}{\varepsilon}\ln\frac{1+\varepsilon}{1-\varepsilon}$, where $\varepsilon=\frac{\sqrt{a^2-b^2}}{a}$ (eccentricity of ellipse). **1723.** a) $\frac{64\pi a^2}{3}$; b) $16\pi^2 a^2$; c) $\frac{32}{3}\pi a^2$.

1724. $\frac{128}{5}\pi a^2$. 1725. $2\pi a^2(2-\sqrt{2})$. 1726. $\frac{128}{5}\pi a^2$. 1727. $M_X=\frac{b}{2}\sqrt{a^2+b^2}$; $M_Y=\frac{a}{2}\sqrt{a^2+b^2}$. 1728. $M_a=\frac{ab^2}{2}$; $M_b=\frac{a^2b}{2}$. 1729. $M_X=M_Y=\frac{a^3}{6}$; $\bar{x}=\bar{y}=\frac{a}{3}$. 1730. $M_X=M_Y=\frac{3}{5}a^2$; $\bar{x}=\bar{y}=\frac{2}{5}a$. 1731. $2\pi a^2$. 1732. $x=0$; $\bar{y}=\frac{a}{4}\frac{2+\sinh 2}{\sinh 1}$. 1733. $\bar{x}=\frac{a\sin\alpha}{\alpha}$; $\bar{y}=0$. 1734. $\bar{x}=\pi a$; $\bar{y}=\frac{4}{3}a$. 1735. $\bar{x}=\frac{4a}{3\pi}$; $\bar{y}=\frac{4b}{3\pi}$. 1736. $\bar{x}=\bar{y}=\frac{9}{20}$. 1737. $\bar{x}=\pi a$; $\bar{y}=\frac{5}{6}a$. 1738. $\left(0, 0, \frac{a}{2}\right)$. **Solution.** Divide the hemisphere into elementary spherical slices of area $d\sigma$ by horizontal planes. We have $d\sigma=2\pi a\,dz$, where dz is the altitude of a slice. Whence $\bar{z}=\frac{2\pi\int_0^a az\,dz}{2\pi a^2}=\frac{a}{2}$. Due to symmetry, $\bar{x}=\bar{y}=0$. 1739. At a distance of $\frac{3}{4}$ altitude from the vertex of the cone. **Solution.** Partition the cone into elements by planes parallel to the base. The mass of an elementary layer (slice) is $dm_i=\gamma\pi\varrho^2\,dz$, where γ is the density, z is the distance of the cutting plane from the vertex of the cone, $\varrho=\frac{r}{h}z$. Whence

$$\bar{z}=\frac{\pi\int_0^h \frac{r^2}{h^2}z^3\,dz}{\frac{1}{3}\pi r^2 h}=\frac{3}{4}h.$$

1740. $\left(0;\ 0;\ +\frac{3}{8}a\right)$. **Solution.** Due to symmetry, $\bar{x}=\bar{y}=0$. To determine $\bar{z}$ we partition the hemisphere into elementary layers (slices) by planes parallel to the horizontal plane. The mass of such an elementary layer $dm=\gamma\pi r^2\,dz$, where γ is the density, z is the distance of the cutting plane from the base of the hemisphere, $r=\sqrt{a^2-z^2}$ is the radius of a cross-section. We have: $\bar{z}=\frac{\pi\int_0^a (a^2-z^2)\,z\,dz}{\frac{2}{3}\pi a^3}=\frac{3}{8}a$. 1741. $I=\pi a^3$.

1742. $I_a=\frac{1}{3}ab^3$; $I_b=\frac{1}{3}a^3b$. 1743. $I=\frac{4}{15}hb^3$. 1744. $I_a=\frac{1}{4}\pi ab^3$; $I_b=\frac{1}{4}\pi a^3b$. 1745. $I=\frac{1}{2}\pi(R_2^4-R_1^4)$. **Solution.** We partition the ring into elementary concentric circles. The mass of each such element $dm=\gamma 2\pi r\,dr$ and the moment of inertia $I=2\pi\int_{R_1}^{R_2} r^3\,dr=\frac{1}{2}\pi(R_2^4-R_1^4)$; $(\gamma=1)$. 1746. $I=\frac{1}{10}\pi R^4 H\gamma$. **Solution.** We partition the cone into elementary cylindrical tubes parallel to the axis of the cone. The volume of each such elementary tube is $dV=2\pi rh\,dr$, where r is the radius of the tube (the distance to the axis of the cone), $h=H\left(1-\frac{r}{R}\right)$ is the altitude of the tube; then the moment of inertia $I=\gamma\int\limits_0^R 2\pi H\left(1-\frac{r}{R}\right)r^3\,dr=\frac{\gamma\pi R^4H}{10}$, where γ is the density of the cone. **1747.** $I=\frac{2}{5}Ma^2$. **Solution.** We partition the sphere into elementary cylindrical tubes, the axis of which is the given diameter. An elementary volume $dV=2\pi rh\,dr$, where r is the radius of a tube, $h=2a\sqrt{1-\frac{r^2}{a^2}}$ is its altitude. Then the moment of inertia $I=4\pi a\gamma\int\limits_0^a\sqrt{1-\frac{r^2}{a^2}}\,r^3dr=\frac{8}{15}\pi a^5\gamma$, where γ is the density of the sphere, and since the mass $M=\frac{4}{3}\pi a^3\gamma$, it follows that $J=\frac{2}{5}Ma^2$. **1748.** $V=2\pi^2a^2b$; $S=4\pi^2ab$. **1749.** a) $\bar{x}=\bar{y}=\frac{2}{5}a$; b) $\bar{x}=\bar{y}=\frac{9}{10}p$. **1750.** a) $\bar{x}=0$, $\bar{y}=\frac{4}{3}\frac{r}{\pi}$ **Hint.** The coordinate axes are chosen so that the x-axis coincides with the diameter and the origin is the centre of the circle; b) $\bar{x}=\frac{h}{3}$ **Solution.** The volume of the solid — a double cone obtained from rotating a triangle about its base, is equal to $V=\frac{1}{3}\pi bh^2$, where b is the base, h is the altitude of the triangle. By the Guldin theorem, the same volume $V=2\pi\bar{x}\frac{1}{2}bh$, where $\bar{x}$ is the distance of the centre of gravity from the base. Whence $\bar{x}=\frac{h}{3}$ **1751.** $v_0t-\frac{gt^2}{2}$. **1752.** $\frac{c^2}{2g}\ln\left(1+\frac{v_0^2}{c^2}\right)$. **1753.** $x=\frac{v_0}{\omega}\sin\omega t$; $v_{av}=\frac{2}{\pi}v_0$ **1754.** $S=10^4$ m. **1755.** $v=\frac{A}{b}\ln\left(\frac{a}{a-bt}\right)$; $h=\frac{A}{b^2}\times\left[bt_1-(a-bt_1)\ln\frac{a}{a-bt_1}\right]$. **1756.** $A=\frac{\pi\gamma}{2}R^2H^2$ **Hint.** The elementary force (force of gravity) is equal to the weight of water in the volume of a layer of thickness dx, that is, $dF=\gamma\pi R^2\,dx$, where γ is the weight of unit volume of water. Hence, the elementary work of a force $dA=\gamma\pi R^2(H-x)\,dx$, where x is the water level. **1757.** $A=\frac{\pi}{12}\gamma R^2H^2$. **1758.** $A=\frac{\pi\gamma}{4}R^4TM\approx 0.79\cdot 10^3=0.79\cdot 10^7$ kgm. **1759.** $A=\gamma\pi R^3H$. **1760.** $A=\frac{mgh}{1+\frac{h}{R}}$; $A_\infty=mgR$. **Solution.** The force acting on a mass m is equal to $F=k\frac{mM}{r^2}$, where r is the distance from the centre of the earth. Since for $r=R$ we have $F=mg$, it follows that $kM=gR^2$. The sought-for work will have the form $A=\int\limits_R^{R+h}k\frac{mM}{r^2}\,dr=kmM\left(\frac{1}{R}-\frac{1}{R+h}\right)=\frac{mgh}{1+\frac{h}{R}}$. When $h=\infty$ we have $A_\infty=mgR$. **1761.** $1.8\cdot 10^4$ ergs. **Solution.**

The force of interaction of charges is $F=\frac{e_0e_1}{x^2}$ dynes. Consequently, the work performed in moving charge e_1 from point x_1 to x_2 is $A=e_0e_1\int\limits_{x_1}^{x_2}\frac{dx}{x^2}=$ $=e_0e_1\left(\frac{1}{x_1}-\frac{1}{x_2}\right)=1.8\cdot10^4$ ergs. **1762.** $A=800\ \pi\ln 2$ kgm. **Solution.** For an isothermal process, $pv=p_0v_0$. The work performed in the expansion of a gas from volume v_0 to volume v_1 is $A=\int\limits_{v_0}^{v_1}p\,dv=p_0v_0\ln\frac{v_1}{v_0}$. **1763.** $A\approx 15{,}000$ kgm.

Solution. For an adiabatic process, the Poisson law $pv^k=p_0v_0^k$, where $k\approx 1.4$, holds true. Hence $A=\int\limits_{v_0}^{v_1}\frac{p_0v_0^k}{v^k}dv=\frac{p_0v_0}{k-1}\left[1-\left(\frac{v_0}{v_1}\right)^{k-1}\right]$.

1764. $A=\frac{4}{3}\pi\mu Pa$. **Solution.** If a is the radius of the base of a shaft, then the pressure on unit area of the support $p=\frac{P}{\pi a^2}$. The frictional force of a ring of width dr, at a distance r from the centre, is $\frac{2\mu P}{a^2}rdr$. The work performed by frictional forces on a ring in one complete revolution is $dA=\frac{4\pi\mu P}{a^2}r^2dr$. Therefore, the complete work $A=\frac{4\pi\mu P}{a^2}\times\int\limits_0^a r^2\,dr=\frac{4}{3}\pi\mu Pa$.

1765. $\frac{1}{4}MR^2\omega^2$. **Solution.** The kinetic energy of a particle of the disk $dK=\frac{mv^2}{2}=\frac{\varrho r^2\omega^2}{2}d\sigma$, where $d\sigma$ is an element of area, r is the distance of it from the axis of rotation, ϱ is the surface density, $\varrho=\frac{M}{\pi R^2}$. Thus, $dK=\frac{M\omega^2}{2\pi R^2}r^2\,d\sigma$. Whence $K=\frac{M\omega^2}{R^2}\int\limits_0^R r^3\,dr=\frac{MR^2\omega^2}{4}$ **1766.** $K=\frac{3}{20}\times MR^2\omega^2$.

1767. $K=\frac{M}{5}R^2\omega^2=2.3\cdot10^8$ kgm. **Hint.** The amount of work required is equal to the reserve of kinetic energy. **1768.** $p=\frac{bh^2}{6}$. **1769.** $P=\frac{(a+2b)\,h^2}{6}\approx 11.3\cdot10^8\ T$ **1770.** $P=ab\gamma\pi h$. **1771.** $P=\frac{\pi R^2H}{3}$ (the vertical component is directed upwards). **1772.** $533\frac{1}{3}$ gm **1773.** 99.8 cal. **1774.** $M=\frac{hb^2p}{2}$ gf cm. **1775.** $\frac{kMm}{a(a+l)}$ (k is the gravitational constant). **1776.** $\frac{\pi pa^4}{8\mu l}$. **Solution.** $Q=\int\limits_0^a v2\pi r\,dr=\frac{2\pi p}{4\mu l}\int\limits_0^a(a^2-r^2)\,r\,dr=$ $=\frac{\pi p}{2\mu l}\left[\frac{a^2r^2}{2}-\frac{r^4}{4}\right]_0^a=\frac{\pi pa^4}{8\mu l}$. **1777.** $Q=\int\limits_0^{2b}v_a dy=\frac{2}{3}p\frac{ab^3}{\mu l}$ **Hint.** Draw the x-axis along the large lower side of the rectangle, and the y-axis, perpendicular to it in the middle. **1778. Solution.** $S=\int\limits_{v_1}^{v_2}\frac{1}{a}\,dv$; on the other hand, $\frac{dv}{dt}=a$, whence $dt=\frac{1}{a}\,dv$, and consequently, the acceleration time is $t=\int\limits_{v_1}^{v_2}\frac{dv}{a}=S$.

1779. $M_x=-\int\limits_0^x\frac{Q}{l}(x-t)\,dt+\frac{Q}{2}x=-\frac{Q}{l}\left[xt-\frac{t^2}{2}\right]_0^x+\frac{Q}{2}x=\frac{Qx}{2}\left(1-\frac{x}{l}\right)$.

1780. $M_x=-\int\limits_0^x(x-t)\,kt\,dt+Ax=\frac{kx}{6}(l^2-x^2)$. **1781.** $Q=0.12\,TRI_0^2$ cal. **Hint.** Use the Joule-Lenz law.

Chapter VI

1782. $V=\frac{2}{3}(y^2-x^2)\,x$. **1783.** $S=\frac{2}{3}(x+y)\sqrt{4z^2+3(x-y)^2}$. **1784.** $f\left(\frac{1}{2},\ 3\right)=\frac{5}{3}$; $f(1,\ -1)=-2$. **1785.** $\frac{y^2-x^2}{2xy}$, $\frac{x^2-y^2}{2xy}$, $\frac{y^2-x^2}{2xy}$, $\frac{2xy}{x^2-y^2}$. **1786.** $f(x,\ x^2)=1+x-x^2$. **1787.** $z=\frac{R^4}{1-R^2}$. **1788.** $f(x)=\frac{\sqrt{1+x^2}}{x}$. **Hint.** Represent the given function in the form $f\left(\frac{y}{x}\right)=\sqrt{\left(\frac{x}{y}\right)^2+1}$ and replace $\frac{y}{x}$ by x. **1789.** $f(x,\ y)=\frac{x^2-xy}{2}$. **Solution.** Designate $x+y=u$, $x-y=v$. Then $x=\frac{u+v}{2}$, $y=\frac{u-v}{2}$; $f(u,\ v)=\frac{u+v}{2}\cdot\frac{u-v}{2}+\left(\frac{u-v}{2}\right)^2=\frac{u^2-uv}{2}$. It remains to name the arguments u and v, x and y. **1790.** $f(u)=u^2+2u$; $z=x-1+\sqrt{y}$. **Hint.** In the identity $x=1+f(\sqrt{x}-1)$ put $\sqrt{x}-1=u$; then $x=(u+1)^2$ and, hence, $f(u)=u^2+2u$. **1791.** $f(y)=\sqrt{1+y^2}$; $z=\sqrt{x^2+y^2}$ **Solution.** When $x=1$ we have the identity $\sqrt{1+y^2}=1\cdot f\left(\frac{y}{1}\right)$, i. e., $f(y)=\sqrt{1+y^2}$. Then $f\left(\frac{y}{x}\right)=\sqrt{1+\left(\frac{y}{x}\right)^2}$ and $z=x\sqrt{1+\left(\frac{y}{x}\right)^2}=\sqrt{x^2+y^2}$. **1792.** a) Single circle with centre at origin, including the circle ($x^2+y^2\leqslant 1$); b) bisector of quadrantal angle $y=x$; c) half-plane located above the straight line $x+y=0$ ($x+y>0$); d) strip contained between the straight lines $y=\pm 1$, including these lines ($-1\leqslant y\leqslant 1$); e) a square formed by the segments of the straight lines $x=\pm 1$ and $y=\pm 1$, including its sides ($-1\leqslant x\leqslant 1$, $-1\leqslant y\leqslant 1$); f) part of the plane adjoining the x-axis and contained between the straight lines $y=\pm x$, including these lines and excluding the coordinate origin ($-x\leqslant y\leqslant x$ when $x>0$, $x\leqslant y\leqslant -x$ when $x<0$); g) two strips $x\geqslant 2$, $-2\leqslant y\leqslant 2$ and $x\leqslant -2$, $-2\leqslant y\leqslant 2$; h) the ring contained between the circles $x^2+y^2=a^2$ and $x^2+y^2=2a^2$, including the boundaries; i) strips $2n\pi\leqslant x\leqslant(2n+1)\pi$, $y\geqslant 0$ and $(2n+1)\pi\leqslant x\leqslant(2n+2)\pi$, $y\leqslant 0$, where n is an integer; j) that part of the plane located above the parabola $y=-x^2\ (x^2+y>0)$; k) the entire xy-plane; l) the entire xy-plane, with the exception of the coordinate origin; m) that part of the plane located above the parabola $y^2=x$ and to the right of the y-axis, including the points of the y-axis and excluding the points of the parabola $(x\geqslant 0,\ y>\sqrt{x})$; n) the entire place except points of the straight lines $x=1$ and $y=0$; o) the family of concentric circles $2\pi k\leqslant x^2+y^2\leqslant\pi(2k+1)$ $(k=0,\ 1,\ 2,\ \ldots)$. **1793.** a) First octant (including boundary); b) First, Third, Sixth and Eighth octants (excluding the boundary); c) a cube bounded by the planes $x=\pm 1$, $y=\pm 1$ and $z=\pm 1$, including its faces; d) a sphere of radius 1 with centre at the origin, including its surface **1794.** a) a plane; the level lines are straight lines parallel to the straight line $x+y=0$; b) a paraboloid of revolution; the level lines are concentric circles with centre at the origin; c) a hyperbolic paraboloid; the level lines are equilateral hyperbolas; d) second-order cone; the level lines are equilateral hyperbolas; e) a parabolic cylinder, the generatrices of which are parallel to the straight line $x+y+1=0$; the level lines are parallel lines; f) the lateral surface of a quadrangular pyramid; the level lines are the outlines of squares; g) level lines are parabolas $y=Cx^2$; h) the level lines are parabolas $y=C\sqrt{x}$; i) the level lines are the circles $C(x^2+y^2)=2x$. **1795.** a) Parabolas $y=C-x^2\ (C>0)$; b) hyperbolas $xy=C\ (|C|\leqslant 1)$; c) circles $x^2+y^2=C^2$; d) straight lines $y=ax+C$; e) straight lines $y=Cx\ (x\neq 0)$. **1796.** a) Planes parallel to the plane $x+y+z=0$; b) concentric spheres with centre at origin; c) for $u>0$, one-sheet hyperboloids of revolution about the z-axis; for $u<0$, two-sheet hyperboloids of revolution about the same axis; both families of surfaces are divided by the cone $x^2+y^2-z^2=0$ $(u=0)$. **1797.** a) 0; b) 0;c) 2; d) e^k; e) limit does not exist; f) limit does not exist. **Hint.** In Item(b) pass to polar coordinates In Items (e) and (f), consider the variation of x and y along the straight lines $y=kx$ and show that the given expression may tend to different limits, depending on the choice of k. **1798.** Continuous. **1799.** a) Discontinuity at $x=0$, $y=0$; b) all points of the straight line $x=y$ (line of discontinuity); c) line of discontinuity is the circle $x^2+y^2=1$; d) the lines of discontinuity are the coordinate axes. **1800** **Hint.** Putting $y=y_1=\text{const}$, we get the function $\varphi_1(x)=\frac{2xy_1}{x^2+y_1^2}$, which is continuous everywhere, since for $y_1\neq 0$ the denominator $x^2+y_1^2\neq 0$, and when $y_1=0$, $\varphi_1(x)\equiv 0$. Similarly, when $x=x_1=\text{const}$, the function $\varphi_2(y)=\frac{2x_1y}{x_1^2+y^2}$ is everywhere continuous. From the set of variables x, y, the function z is discontinuous at the point (0, 0) since there is no $\lim\limits_{\substack{x\to 0\\ y\to 0}} z$. Indeed, passing to polar coordinates $(x=r\cos\varphi, y=r\sin\varphi)$, we get $z=\sin 2\varphi$, whence it is evident that if $x\to 0$ and $y\to 0$ in such manner that $\varphi=\text{const}$ $(0\leqslant\varphi\leqslant 2\pi)$, then $z\to\sin 2\varphi$. Since these limiting values of the function z depend on the direction of φ, it follows that z does not have a limit as $x\to 0$ and $y\to 0$. **1801.** $\frac{\partial z}{\partial x}=3(x^2-ay)$, $\frac{\partial z}{\partial y}=3(y^2-ax)$. **1802.** $\frac{\partial z}{\partial x}=\frac{2y}{(x+y)^2}$, $\frac{\partial z}{\partial y}=-\frac{2x}{(x+y)^2}$. **1803.** $\frac{\partial z}{\partial x}=-\frac{y}{x^2}$, $\frac{\partial z}{\partial y}=\frac{1}{x}$. **1804.** $\frac{\partial z}{\partial x}=\frac{x}{\sqrt{x^2-y^2}}$, $\frac{\partial z}{\partial y}=-\frac{y}{\sqrt{x^2-y^2}}$. **1805.** $\frac{\partial z}{\partial x}=\frac{y^2}{(x^2+y^2)^{3/2}}$, $\frac{\partial z}{\partial y}=-\frac{xy}{(x^2+y^2)^{3/2}}$. **1806.** $\frac{\partial z}{\partial x}=\frac{1}{\sqrt{x^2+y^2}}$, $\frac{\partial z}{\partial y}=\frac{y}{\sqrt{x^2+y^2}\,(x+\sqrt{x^2+y^2})}$. **1807.** $\frac{\partial z}{\partial x}=-\frac{y}{x^2+y^2}$, $\frac{\partial z}{\partial y}=\frac{x}{x^2+y^2}$. **1808.** $\frac{\partial z}{\partial x}=yx^{y-1}$,

$\frac{\partial z}{\partial y}=x^y \ln x$. 1809. $\frac{\partial z}{\partial x}=-\frac{y}{x^2}e^{\sin\frac{y}{x}}\cos\frac{y}{x}$, $\frac{\partial z}{\partial y}=\frac{1}{x}e^{\sin\frac{y}{x}}\cos\frac{y}{x}$. 1810. $\frac{\partial z}{\partial x}=$ $=\frac{xy^2\sqrt{2x^2-2y^2}}{|y|(x^4-y^4)}$, $\frac{\partial z}{\partial y}=-\frac{yx^2\sqrt{2x^2-2y^2}}{|y|(x^4-y^4)}$. 1811. $\frac{\partial z}{\partial x}=\frac{1}{\sqrt{y}}\cot\frac{x+a}{\sqrt{y}}$, $\frac{\partial z}{\partial y}=-\frac{x+a}{2y\sqrt{y}}\cot\frac{x+a}{\sqrt{y}}$. 1812. $\frac{\partial u}{\partial x}=yz(xy)^{z-1}$, $\frac{\partial u}{\partial y}=xz(xy)^{z-1}$, $\frac{\partial u}{\partial z}=(xy)^z\ln(xy)$.

1813. $\frac{\partial u}{\partial x}=yz^{xy}\ln z$, $\frac{\partial u}{\partial y}=xz^{xy}\ln z$, $\frac{\partial u}{\partial z}=xyz^{xy-1}$. 1814. $f'_x(2, 1)=\frac{1}{2}$, $f'_y(2, 1)=0$. 1815. $f'_x(1, 2, 0)=1$, $f_y(1, 2, 0)=\frac{1}{2}$, $f^z(1, 2, 0)=\frac{1}{2}$. 1820. $-\frac{x}{(x^2+y^2+z^2)^{3/2}}$. 1821. r. 1826. $z=\arctan\frac{y}{x}+\varphi(x)$. 1827. $z=\frac{x^2}{2}+$ $+y^2\ln x+\sin y-\frac{1}{2}$. 1828. 1) $\tan\alpha=4$, $\tan\beta=\infty$, $\tan\gamma=\frac{1}{4}$; 2) $\tan\alpha=\infty$, $\tan\beta=4$, $\tan\gamma=\frac{1}{4}$. 1829. $\frac{\partial S}{\partial a}=\frac{1}{2}h$, $\frac{\partial S}{\partial b}=\frac{1}{2}h$, $\frac{\partial S}{\partial h}=\frac{1}{2}(a+b)$. 1830. **Hint.** Check to see that the function is equal to zero over the entire x-axis and the entire y-axis, and take advantage of the definition of partial derivatives. Be convinced that $f'_x(0, 0)=f'_y(0, 0)=0$. 1831. $\Delta f=4\Delta x+\Delta y+2\Delta x^2+$ $+2\Delta x\,\Delta y+\Delta x^2\,\Delta y$; $df=4dx+dy$; a) $\Delta f-df=8$; b) $\Delta f-df=0.062$. 1833. $dz=3(x^2-y)\,dx+3(y^2-x)\,dy$. 1834. $dz=2xy^3dx+3x^2y^2dy$. 1835. $dz=$ $=\frac{4}{(x^2+y^2)^2}(xy^2dx-x^2y\,dy)$. 1836. $dz=\sin 2x\,dx-\sin 2y\,dy$. 1837. $dz=y^2x^{y-1}dx+$ $+x^y(1+y\ln x)\,dy$. 1838. $dz=\frac{2}{x^2+y^2}(x\,dx+y\,dy)$. 1839. $df=\frac{1}{x+y}\left(dx-\frac{x}{y}dy\right)$. 1840. $dz=0$. 1841. $dz=\frac{2}{x\sin\frac{2y}{x}}\left(dy-\frac{y}{x}dx\right)$. 1842. $df(1, 1)=dx-2dy$.

1843. $du=yz\,dx+zx\,dy+xy\,dz$. 1844. $du=\frac{1}{\sqrt{x^2+y^2+z^2}}(x\,dx+y\,dy+z\,dz)$. 1845. $du=\left(xy+\frac{x}{y}\right)^{z-1}\left[\left(y+\frac{1}{y}\right)z\,dx+\left(1-\frac{1}{y^2}\right)xz\,dy+\left(xy+\frac{x}{y}\right)\ln\times\right.$ $\left.\times\left(xy+\frac{x}{y}\right)dz\right]$. 1846. $du=\frac{z^2}{x^2y^2+z^4}\left(y\,dx+x\,dy-\frac{2xy}{z}dz\right)$. 1847. $df(3, 4, 5)=$ $=\frac{1}{25}(5dz-3dx-4dy)$. 1848. $dl=0.062$ cm; $\Delta l=0.065$ cm. 1849. 75 cm^3 (relative to inner dimensions). 1850. $\frac{1}{8}$ cm. **Hint.** Put the differential of the area of the sector equal to zero and find the differential of the radius from that. 1851. a) 1.00; b) 4.998, c) 0.273. 1853. Accurate to 4 metres (more exactly, 4.25 m). 1854. $\pi\frac{\alpha g-\beta l}{g\sqrt{lg}}$. 1855. $d\alpha=\frac{1}{\varrho}(dy\cos\alpha-dx\sin\alpha)$. 1856. $\frac{dz}{dt}=$ $=\frac{e^t(t\ln t-1)}{t\ln^2 t}$. 1857. $\frac{du}{dt}=\frac{t}{\sqrt{y}}\cot\frac{x}{\sqrt{y}}\left(6-\frac{x}{2y^2}\right)$. 1858. $\frac{du}{dt}=2t\ln t\tan t+$ $+\frac{(t^2+1)\tan t}{t}+\frac{(t^2+1)\ln t}{\cos^2 t}$. 1859. $\frac{du}{dt}=0$. 1860. $\frac{dz}{dx}=(\sin x)^{\cos x}(\cos x\cot x-$

$-\sin x \ln \sin x)$. **1861.** $\frac{\partial z}{\partial x}=-\frac{y}{x^2+y^2}$; $\frac{dz}{dx}=\frac{1}{1+x^2}$. **1862.** $\frac{\partial z}{\partial x}=yx^{y-1}$; $\frac{dz}{dx}=x^y\left[\varphi'(x)\ln x+\frac{y}{x}\right]$. **1863.** $\frac{\partial z}{\partial x}=2xf'_u(u, v)+ye^{xy}f'_v(u, v)$; $\frac{\partial z}{\partial y}=-2yf'_u(u, v)+xe^{xy}f'_v(u, v)$. **1864** $\frac{\partial z}{\partial u}=0$, $\frac{\partial z}{\partial v}=1$. **1865.** $\frac{\partial z}{\partial x}=y\left(1-\frac{1}{x^2}\right)f'\left(xy+\frac{y}{x}\right)$; $\frac{\partial z}{\partial y}=\left(x+\frac{1}{x}\right)f'\left(xy+\frac{y}{x}\right)$. **1867.** $\frac{du}{dx}=f'_x(x, y, z)+\varphi'(x)f'_y(x, y, z)+f'_z(x, y, z)[\psi'_x(x, y)+\psi'_y(x, y)\varphi'(x)]$. **1873.** The perimeter increases at a rate of 2 m/sec, the area increases at a rate of 70 m²/sec. **1874.** $\frac{1+2t^2+3t^4}{\sqrt{1+t^2+t^4}}$. **1875.** $20\sqrt{5-2\sqrt{2}}$ km/hr. **1876.** $-\frac{9\sqrt{3}}{2}$. **1877.** 1. **1878.** $\frac{\sqrt{2}}{2}$. **1879.** $-\frac{\sqrt{3}}{3}$. **1880.** $\frac{68}{13}$. **1881.** $\frac{\cos\alpha+\cos\beta+\cos\gamma}{3}$. **1882.** a) (2, 0); b) (0, 0); and (1, 1); c) (7, 2, 1). **1884.** $9i-3j$. **1885.** $\frac{1}{4}(5i-3j)$. **1886.** $6i+3j+2k$. **1887.** $|\text{grad}\, u|=6$; $\cos\alpha=\frac{2}{3}$, $\cos\beta=-\frac{2}{3}$, $\cos\gamma=\frac{1}{3}$. **1888.** $\cos\varphi=\frac{3}{\sqrt{10}}$. **1889.** $\tan\varphi\approx 8.944$; $\varphi\approx 83°37'$. **1891.** $\frac{\partial^2 z}{\partial x^2}=\frac{abcy^2}{(b^2x^2+a^2y^2)^{3/2}}$; $\frac{\partial^2 z}{\partial x\,\partial y}=-\frac{abcxy}{(b^2x^2+a^2y^2)^{3/2}}$; $\frac{\partial^2 z}{\partial y^2}=\frac{abcx^2}{(b^2x^2+a^2y^2)^{3/2}}$. **1892** $\frac{\partial^2 z}{\partial x^2}=\frac{2(y-x^2)}{(x^2+y)^2}$; $\frac{\partial^2 z}{\partial x\,\partial y}=-\frac{2x}{(x^2+y)^2}$; $\frac{\partial^2 z}{\partial y^2}=-\frac{1}{(x^2+y)^2}$. **1893.** $\frac{\partial^2 z}{\partial x\,\partial y}=\frac{xy}{(2xy+y^2)^{3/2}}$. **1894.** $\frac{\partial^2 z}{\partial x\,\partial y}=0$. **1895.** $\frac{\partial^2 r}{\partial x^2}=\frac{r^2-x^2}{r^3}$ **1896.** $\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial z^2}=0$; $\frac{\partial^2 u}{\partial x\,\partial y}=\frac{\partial^2 u}{\partial y\,\partial z}=\frac{\partial^2 u}{\partial z\,\partial x}=1$. **1897.** $\frac{\partial^3 u}{\partial x\,\partial y\,\partial z}=\alpha\beta\gamma x^{\alpha-1}y^{\beta-1}z^{\gamma-1}$. **1898.** $\frac{\partial^3 z}{\partial x\,\partial y^2}=-x^2y\cos(xy)-2x\sin(xy)$. **1899.** $f''_{x}(0, 0)=m(m-1)$; $f''_{xy}(0, 0)=mn$; $f''_{yy}(0, 0)=n(n-1)$. **1902. Hint.** Using the rules of differentiation and the definition of a partial derivative, verify that $f'_x(x, y)=y\left[\frac{x^2-y^2}{x^2+y^2}+\frac{4x^2y^2}{(x^2+y^2)^2}\right]$ (when $x^2+y^2\neq 0$), $f'_x(0, 0)=0$ and, consequently, that for $x=0$ and for any y, $f'_x(0, y)=-y$. Whence $f''_{xy}(0, y)=-1$; in particular, $f''_{xy}(0, 0)=-1$. Similarly, we find that $f''_{yx}(0, 0)=1$.

1903. $\frac{\partial^2 z}{\partial x^2}=2f'_u(u, v)+4x^2f''_{uu}(u, v)+4xyf''_{uv}(u, v)+y^2f''_{vv}(u, v)$;

$\frac{\partial^2 z}{\partial x\,\partial y}=f'_v(u, v)+4xyf''_{uu}(u, v)+2(x^2+y^2)f''_{uv}(u, v)+xyf''_{vv}(u, v)$;

$\frac{\partial^2 z}{\partial y^2}=2f'_u(u, v)+4y^2f''_{uu}(u, v)+4xyf''_{uv}(u, v)+x^2f''_{vv}(u, v)$.

1904. $\frac{\partial^2 u}{\partial x^2}=f''_{xx}+2f''_{xz}\varphi'_x+f''_{zz}(\varphi'_x)^2+f'_z\varphi''_{xx}$.

1905. $\frac{\partial^2 z}{\partial x^2}=f''_{uu}(\varphi'_x)^2+2f''_{uv}\varphi'_x\psi'_x+f''_{vv}(\psi'_x)^2+f'_u\varphi''_{xx}+f'_v\psi''_{xx}$;

$\frac{\partial^2 z}{\partial x\,\partial y}=f''_{uu}\varphi'_x\varphi'_y+f''_{uv}(\varphi'_x\psi'_y+\psi'_x\varphi'_y)+f''_{vv}\psi'_x\psi'_y+f'_u\varphi''_{xy}+f'_v\psi''_{xy}$;

$\frac{\partial^2 z}{\partial y^2}=f''_{uu}(\varphi'_y)^2+2f''_{uv}\varphi'_y\psi'_y+f''_{vv}(\psi'_y)^2+f'_u\varphi''_{yy}+f'_v\psi''_{yy}$.

1914. $u(x, y)=\varphi(x)+\psi(y)$. **1915.** $u(x, y)=x\varphi(y)+\psi(y)$. **1916.** $d^2z=e^{xy}\times\times[(y\,dx+x\,dy)^2+2dx\,dy]$. **1917.** $d^2u=2(x\,dy\,dz+y\,dz\,dx+z\,dx\,dy)$. **1918.** $d^2z=4\varphi''(t)(x\,dx+y\,dy)^2+2\varphi'(t)(dx^2+dy^2)$. **1919.** $dz=\left(\frac{x}{y}\right)^{xy}\times\times\left(y\ln\frac{ex}{y}dx+x\ln\frac{x}{ey}dy\right)$; $d^2z=\left(\frac{x}{y}\right)^{xy}\left[\left(y^2\ln^2\frac{ex}{y}+\frac{y}{x}\right)dx^2+\right.$ $\left.+2\left(xy\ln\frac{ex}{y}\ln\frac{x}{ey}+\ln\frac{x}{y}\right)dx\,dy+\left(x^2\ln^2\frac{x}{ey}-\frac{x}{y}\right)dy^2\right]$.

1920. $d^2z=a^2f''_{uu}(u, v)\,dx^2+2abf''_{uv}(u, v)\,dx\,dy+b^2f''_{vv}(u, v)\,dy^2$.

1921. $d^2z=(ye^xf'_v+e^{2y}f''_{uu}+2ye^{x+y}f''_{uv}+y^2e^{2x}f''_{vv})\,dx^2+$ $+2(e^yf'_u+e^xf'_v+xe^{2y}f''_{uu}+e^{x+y}(1+xy)f''_{uv}+ye^{2x}f''_{vv})\,dx\,dy+$ $+\left(xe^yf'_u+x^2e^{2y}f''_{uu}+2xe^{x+y}f''_{uv}+e^{2x}f''_{vv}\right)dy^2$. **1922.** $d^3z=e^x(\cos y\,dx^3-$ $-3\sin y\,dx^2\,dy-3\cos y\,dx\,dy^2+\sin y\,dy^3)$. **1923.** $d^3z=-y\cos x\,dx^3-$ $-3\sin x\,dx^2\,dy-3\cos y\,dx\,dy^2+x\sin y\,dy^3$. **1924.** $df(1, 2)=0$; $d^2f(1, 2)=$ $=6dx^2+2dx\,dy+4.5\,dy^2$. **1925.** $d^2f(0, 0, 0)=2dx^2+4dy^2+6dz^2-4dx\,dy+$ $+8dx\,dz+4dy\,dz$. **1926.** $xy+C$. **1927.** $x^3y-\frac{y^3}{3}+\sin x+C$. **1928.** $\frac{x}{x+y}+$ $+\ln(x+y)+C$. **1929.** $\frac{1}{2}\ln(x^2+y^2)+2\arctan\frac{x}{y}+C$. **1930.** $\frac{x}{y}+C$.

1931. $\sqrt{x^2+y^2}+C$. **1932.** $a=-1$, $b=-1$, $z=\frac{x-y}{x^2+y^2}+C$. **1933.** $x^2+y^2+z^2+$ $+xy+xz+yz+C$. **1934.** $x^3+2xy^2+3xz+y^2-yz-2z+C$. **1935.** $x^2yz-3xy^2z+$ $+4x^2y^2+2x+y+3z+C$. **1936.** $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+C$. **1937.** $\sqrt{x^2+y^2+z^2}+C$

1938. $\lambda=-1$. **Hint.** Write the condition of the total differential for the expression $X\,dx+Y\,dy$. **1939.** $f'_x=f'_y$. **1940.** $u=\int_a^{xy} f(z)\,dz+C$. **1941.** $\frac{dy}{dx}=$ $=-\frac{b^2x}{a^2y}$; $\frac{d^2y}{dx^2}=-\frac{b^4}{a^2y^3}$; $\frac{d^3y}{dx^3}=-\frac{3b^6x}{a^4y^5}$. **1942.** The equation defining y is the equation of a pair of straight lines. **1943.** $\frac{dy}{dx}=\frac{y^x\ln y}{1-xy^{x-1}}$. **1944.** $\frac{dy}{dx}=\frac{y}{y-1}$; $\frac{d^2y}{dx^2}=\frac{y}{(1-y)^3}$. **1945.** $\left(\frac{dy}{dx}\right)_{x=1}=3$ or -1; $\left(\frac{d^2y}{dx^2}\right)_{x=1}=8$ or -8.

1946. $\frac{dy}{dx}=\frac{x+ay}{ax-y}$; $\frac{d^2y}{dx^2}=\frac{(a^2+1)(x^2+y^2)}{(ax-y)^3}$. **1947.** $\frac{dy}{dx}=-\frac{y}{x}$; $\frac{d^2y}{dx^2}=\frac{2y}{x^2}$.

1948. $\frac{\partial z}{\partial x}=\frac{x^2-yz}{xy-z^2}$; $\frac{\partial z}{\partial y}=\frac{6y^2-3xz-2}{3(xy-z^2)}$. **1949.** $\frac{\partial z}{\partial x}=\frac{z\sin x-\cos y}{\cos x-y\sin z}$; $\frac{\partial z}{\partial y}=$ $=\frac{x\sin y-\cos z}{\cos x-y\sin z}$. **1950.** $\frac{\partial z}{\partial x}=-1$; $\frac{\partial z}{\partial y}=\frac{1}{2}$. **1951.** $\frac{\partial z}{\partial x}=-\frac{c^2x}{a^2z}$; $\frac{\partial z}{\partial y}=-\frac{c^2y}{b^2z}$;

$\frac{\partial^2 z}{\partial x^2}=-\frac{c^4(b^2-y^2)}{a^2b^2z^3}$; $\frac{\partial^2 z}{\partial x\,\partial y}=-\frac{c^4xy}{a^2b^2z^3}$; $\frac{\partial^2 z}{\partial y^2}=-\frac{c^4(a^2-x^2)}{a^2b^2z^3}$. **1953.** $\frac{dz}{dx}=$ $=\frac{\begin{vmatrix}\varphi'_x & \varphi'_y\\ \psi'_x & \psi'_y\end{vmatrix}}{\psi'_y}$. **1954.** $dz=-\frac{x}{z}dx-\frac{y}{z}dy$; $d^2z=\frac{y^2-a^2}{z^3}dx^2-2\frac{xy}{z^3}dx\,dy+$ $+\frac{x^2-a^2}{z^3}dy^2$. **1955.** $dz=0$; $d^2z=\frac{4}{15}(dx^2+dy^2)$. **1956.** $dz=\frac{z}{1-z}(dx+dy)$; $d^2z=\frac{z}{(1-z)^3}(dx^2+2dx\,dy+dy^2)$. **1961.** $\frac{dy}{dx}=\infty$; $\frac{dz}{dx}=\frac{1}{5}$; $\frac{d^2y}{dx^2}=\infty$; $\frac{d^2z}{dx^2}=\frac{4}{25}$. **1962.** $dy=\frac{y(z-x)}{x(y-z)}dx$; $dz=\frac{z(x-y)}{x(y-z)}dx$; $d^2y=-d^2z=-\frac{a}{x^3(y-z)^3}\times$ $\times[(x-y)^2+(y-z)^2+(z-x)^2]\,dx^2$. **1963.** $\frac{\partial u}{\partial x}=\frac{\partial u}{\partial y}=1$; $\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial x\,\partial y}=\frac{\partial^2 u}{\partial y^2}=0$; $\frac{\partial v}{\partial x}=-1$; $\frac{\partial v}{\partial y}=0$; $\frac{\partial^2 v}{\partial x^2}=2$; $\frac{\partial^2 v}{\partial x\,\partial y}=1$; $\frac{\partial^2 v}{\partial y^2}=0$. **1964.** $du=\frac{y}{1+y}dx+$ $+\frac{v}{1+y}dy$; $dv=\frac{1}{1+y}dx-\frac{v}{1+y}dy$; $d^2u=-d^2v=\frac{2}{(1+y)^2}dx\,dy-$ $-\frac{2v}{(1+y)^2}dy^2$. **1965.** $du=\frac{\psi'_v dx-\varphi'_v dy}{\begin{vmatrix}\varphi'_u & \varphi'_v\\ \psi'_u & \psi'_v\end{vmatrix}}$; $dv=\frac{-\psi'_u dx+\varphi'_u dy}{\begin{vmatrix}\varphi'_u & \varphi'_v\\ \psi'_u & \psi'_v\end{vmatrix}}$.

1966. a) $\frac{\partial z}{\partial x}=-\frac{c\sin v}{u}$, $\frac{\partial z}{\partial y}=\frac{c\cos v}{u}$; b) $\frac{\partial z}{\partial x}=\frac{1}{2}(v+u)$, $\frac{\partial z}{\partial y}=\frac{1}{2}(v-u)$; c) $dz=\frac{1}{2e^{2u}}[e^{u-v}(v+u)\,dx+e^{u+v}(v-u)\,dy]$. **1967.** $\frac{\partial z}{\partial x}=F'_r(r,\varphi)\cos\varphi-$ $-F'_\varphi(r,\varphi)\frac{\sin\varphi}{r}$; $\frac{\partial z}{\partial y}=F'_r(r,\varphi)\sin\varphi+F'_\varphi(r,\varphi)\frac{\cos\varphi}{r}$. **1968.** $\frac{\partial z}{\partial x}=$ $=-\frac{c}{a}\cos\varphi\cot\psi$; $\frac{\partial z}{\partial y}=-\frac{c}{b}\sin\varphi\cot\psi$. **1969.** $\frac{d^2y}{dt^2}+\frac{dy}{dt}+y=0$. **1970.** $\frac{d^2y}{dt^2}=0$. **1971.** a) $\frac{d^2x}{dy^2}-2y\frac{dx}{dy}=0$; b) $\frac{d^3x}{dy^3}=0$. **1972.** $\tan\mu=\frac{r}{\frac{dr}{d\varphi}}$.

1973. $K=\frac{r^2+2\left(\frac{dr}{d\varphi}\right)^2-r\frac{d^2r}{d\varphi^2}}{\left[r^2+\left(\frac{dr}{d\varphi}\right)^2\right]^{3/2}}$. **1974.** $\frac{\partial z}{\partial u}=0$. **1975.** $u\frac{\partial z}{\partial u}-z=0$. **1976.** $\frac{\partial^2 u}{\partial r^2}+$ $+\frac{1}{r^2}\frac{\partial^2 u}{\partial\varphi^2}+\frac{1}{r}\frac{\partial u}{\partial r}=0$. **1977.** $\frac{\partial^2 z}{\partial u\,\partial v}=\frac{1}{2u}\frac{\partial z}{\partial v}$. **1978.** $\frac{\partial w}{\partial v}=0$. **1979.** $\frac{\partial^2 w}{\partial v^2}=0$. **1980.** $\frac{\partial^2 w}{\partial u^2}=\frac{1}{2}$. **1981.** a) $2x-4y-z-5=0$; $\frac{x-1}{2}=\frac{y+2}{-4}=\frac{z-5}{-1}$; b) $3x+4y-$ $-6z=0$; $\frac{x-4}{3}=\frac{y-3}{4}=\frac{z-4}{-6}$; c) $x\cos\alpha+y\sin\alpha-R=0$, $\frac{x-R\cos\alpha}{\cos\alpha}=$ $=\frac{y-R\sin\alpha}{\sin\alpha}=\frac{z-R}{0}$. **1982.** $\pm\frac{a^2}{\sqrt{a^2+b^2+c^2}}$; $\pm\frac{b^2}{\sqrt{a^2+b^2+c^2}}$; $\pm\frac{c^2}{\sqrt{a^2+b^2+c^2}}$.

1983. $3x+4y+12z-169=0$. **1985.** $x+4y+6z=\pm 21$ **1986.** $x+y+z=\pm\sqrt{a^2+b^2+c^2}$ **1987** At the points $(1, \pm 1, 0)$, the tangent planes are parallel to the xz-plane; at the points $(0, 0, 0)$ and $(2, 0, 0)$, to the yz-plane. There are no points on the surface at which the tangent plane is parallel to the xy-plane. **1991.** $\frac{\pi}{3}$. **1994.** Projection on the xy-plane: $\begin{cases} z=0 \\ x^2+y^2-xy-1=0. \end{cases}$ Projection on the yz-plane: $\begin{cases} x=0 \\ \frac{3y^2}{4}+z^2-1=0 \end{cases}$ Projection on the xz-plane: $\begin{cases} y=0 \\ \frac{3x^2}{4}+z^2-1=0. \end{cases}$ **Hint.** The line of tangency of the surface with the cylinder projecting this surface on some plane is a locus at which the tangent plane to the given surface is perpendicular to the plane of the projection **1996.** $f(x+h, \quad y+k)=ax^2+2bxy+cy^2+2(ax+by)h+2(bx+cy)k+ah^2+2bhk+ck^2$ **1997.** $f(x, y)=1-(x+2)^2+2(x+2)(y-1)+3(y-1)^2$. **1998.** $\Delta f(x, y)=2h+k+h^2+2hk+h^2k$. **1999.** $f(x, y, z)=(x-1)^2+(y-1)^2+(z-1)^2+2(x-1)(y-1)-(y-1)(z-1)$. **2000.** $f(x+h, \quad y+k, z+l)=f(x, y, z)+2[h(x-y-z)+k(y-x-z)+l(z-x-y)]+f(h, k, l)$. **2001.** $y+xy+\frac{3x^2y-y^3}{3!}$. **2002.** $1-\frac{x^2+y^2}{2!}+\frac{x^4+6x^2y^2+y^4}{4!}$. **2003.** $1+(y-1)+(x-1)(y-1)$. **2004.** $1+[(x-1)+(y+1)]+\frac{[(x-1)+(y+1)]^2}{2!}+\frac{[(x-1)+(y+1)]^3}{3!}$. **2005.** a) $\arctan\frac{1+\alpha}{1-\beta}\approx\frac{\pi}{4}+\frac{1}{2}(\alpha+\beta)-\frac{1}{4}(\alpha^2-\beta^2)$; b) $\sqrt{\frac{(1+\alpha)^m+(1+\beta)^n}{2}}\approx 1+\frac{1}{4}(m\alpha+n\beta)+\frac{1}{32}[(3m^2-4m)\alpha^2-2mn\alpha\beta+(3n^2-4n)\beta^2]$. **2006.** a) 1.0081; b) 0.902. **Hint.** Apply Taylor's formula for the functions: a) $f(x, y)=\sqrt{x}\sqrt[3]{y}$ in the neighbourhood of the point $(1,1)$; b) $f(x, y)=y^x$ in the neighbourhood of the point $(2,1)$. **2007.** $z=1+2(x-1)-(y-1)-8(x-1)^2+10(x-1)(y-1)-3(y-1)^2+\ldots$ **2008.** $z_{\min}=0$ when $x=1$, $y=0$ **2009.** No extremum. **2010.** $z_{\min}=-1$ when $x=1$, $y=0$. **2011.** $z_{\max}=108$ when $x=3$, $y=2$. **2012.** $z_{\min}=-8$ when $x=\sqrt{2}$, $y=-\sqrt{2}$ and when $x=-\sqrt{2}$, $y=\sqrt{2}$. There is no extremum for $x=y=0$. **2013.** $z_{\max}=\frac{ab}{3\sqrt{3}}$ at the points $x=\frac{a}{\sqrt{3}}$, $y=\frac{b}{\sqrt{3}}$ and $x=-\frac{a}{\sqrt{3}}$, $y=-\frac{b}{\sqrt{3}}$; $z_{\min}=-\frac{ab}{3\sqrt{3}}$ at the points $x=\frac{a}{\sqrt{3}}$, $y=-\frac{b}{\sqrt{3}}$ and $x=-\frac{a}{\sqrt{3}}$, $y=\frac{b}{\sqrt{3}}$. **2014.** $z_{\max}=1$ when $x=y=0$. **2015.** $z_{\min}=0$ when $x=y=0$; nonrigorous maximum $\left(z=\frac{1}{e}\right)$ at points of the circle $x^2+y^2=1$. **2016.** $z_{\max}=\sqrt{3}$ when $x=1$, $y=-1$. **2017.** $u_{\min}=-\frac{4}{3}$ when $x=-\frac{2}{3}$, $y=-\frac{1}{3}$, $z=1$. **2018.** $u_{\min}=4$ when $x=\frac{1}{2}$, $y=1$, $z=1$. **2019.** The equation defines two functions, of which one has a maximum ($z_{\max}=8$) when $x=1$, $y=-2$; the other has a minimum ($z_{\min}=-2$) when $x=1$, $y=-2$, at points of the circle $(x-1)^2+(y+2)^2=25$, each of these functions has a boundary extremum ($z=3$). **Hint.** The functions mentioned in the answer are explicitly defined by the equalities

$z=3\pm\sqrt{25-(x-1)^2-(y+2)^2}$ and consequently exist only inside and on the boundary of the circle $(x-1)^2+(y+2)^2=25$, at the points of which both functions assume the value $z=3$. This value is the least for the first function and is the greatest for the second. **2020.** One of the functions defined by the equation has a maximum ($z_{max}=-2$) for $x=-1$, $y=2$, the other has a minimum ($z_{min}=1$) for $x=-1$, $y=2$, both functions have a boundary extremum at the points of the curve $4x^3-4y^2-12x+16y-33=0$. **2021.** $z_{max}=\frac{1}{4}$ for $x=y=\frac{1}{2}$. **2022.** $z_{max}=5$ for $x=1$, $y=2$; $z_{min}=-5$ for $x=-1$, $y=-2$ **2023.** $z_{min}=\frac{36}{13}$ for $x=\frac{18}{13}$, $y=\frac{12}{13}$. **2024.** $z_{max}=\frac{2+\sqrt{2}}{2}$ for $x=\frac{7\pi}{8}+k\pi$, $y=\frac{9\pi}{8}+k\pi$, $z_{min}=\frac{2-\sqrt{2}}{2}$ for $x=\frac{3\pi}{8}+k\pi$, $y=\frac{5\pi}{8}+k\pi$. **2025.** $u_{min}=-9$ for $x=-1$, $y=2$, $z=-2$, $u_{max}=9$ for $x=1$, $y=-2$, $z=2$. **2026.** $u_{max}=a$ for $x=\pm a$, $y=z=0$; $u_{min}=c$ for $x=y=0$ $z=\pm c$. **2027.** $u_{max}=2\cdot4^2\cdot6^3$ for $x=2$, $y=4$, $z=6$. **2028.** $u_{max}=4^4/_{27}$ at the points $\left(\frac{4}{3},\frac{4}{3},\frac{7}{3}\right)$; $\left(\frac{4}{3},\frac{7}{3},\frac{4}{3}\right)$; $\left(\frac{7}{3},\frac{4}{3},\frac{4}{3}\right)$; $u_{min}=4$ at the points (2, 2, 1) (2, 1, 2) (1, 2, 2). **2030.** a) Greatest value $z=3$ for $x=0$, $y=1$; b) smallest value $z=2$ for $x=1$, $y=0$. **2031.** a) Greatest value $z=\frac{2}{3\sqrt{3}}$ for $x=\pm\sqrt{\frac{2}{3}}$, $y=\sqrt{\frac{1}{3}}$; smallest value $z=-\frac{2}{3\sqrt{3}}$ for $x=\pm\sqrt{\frac{2}{3}}$, $y=-\sqrt{\frac{1}{3}}$; b) greatest value $z=1$ for $x=\pm1$, $y=0$; smallest value $z=-1$ for $x=0$, $y=\pm1$. **2032.** Greatest value $z=\frac{3\sqrt{3}}{2}$ for $x=y=\frac{\pi}{3}$ (internal maximum); smallest value $z=0$ for $x=y=0$ (boundary minimum). **2033.** Greatest value $z=13$ for $x=2$, $y=-1$ (boundary maximum); smallest value $z=-2$ for $x=y=1$ (internal minimum) and for $x=0$, $y=-1$ (boundary minimum). **2034.** Cube. **2035.** $\sqrt[3]{2V}$, $\sqrt[3]{2V}$, $\frac{1}{2}\sqrt[3]{2V}$. **2036.** Isosceles triangle. **2037.** Cube. **2038.** $a=\sqrt[4]{a}\cdot\sqrt[4]{a}\cdot\sqrt[4]{a}\cdot\sqrt[4]{a}$. **2039.** $M\left(-\frac{1}{4},\frac{1}{4}\right)$. **2040.** Sides of the triangle are $\frac{3}{4}p$, $\frac{3}{4}p$, and $\frac{p}{2}$. **2041.** $x=\frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}$, $y=\frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}$. **2042.** $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3$. **2043.** The dimensions of the parallelepiped are $\frac{2a}{\sqrt{3}}$, $\frac{2b}{\sqrt{3}}$, $\frac{2c}{\sqrt{3}}$, where a, b, and c are the semi-axes of the ellipsoid. **2044.** $x=y=2\delta+\sqrt[3]{2V}$, $z=\frac{x}{2}$. **2045.** $x=\pm\frac{a}{\sqrt{2}}$, $y=\pm\frac{b}{\sqrt{2}}$ **2046.** Major axis, $2a=6$, minor axis, $2b=2$. **Hint.** The square of the distance of the point (x, y) of the ellipse from its centre (coordinate origin) is equal to x^2+y^2. The problem reduces to finding the extremum of the function x^2+y^2 provided $5x^2+8xy+5y^2=9$. **2047.** The radius of the base of the cylinder is $\frac{R}{2}\sqrt{2+\frac{2}{\sqrt{5}}}$, the altitude $R\sqrt{2-\frac{2}{\sqrt{5}}}$, where R is the radius of the sphere. **2048.** The channel must connect the point of the parabola $\left(\frac{1}{2}, \frac{1}{4}\right)$ with the point of the straight line $\left(\frac{11}{8}, -\frac{5}{8}\right)$; its length is $\frac{7\sqrt{2}}{8}$. **2049.** $\frac{1}{14}\sqrt{2730}$. **2050.** $\frac{\sin\alpha}{\sin\beta}=\frac{v_1}{v_2}$. **Hint.** Obviously, the point M, at which the ray passes from one medium into the other, must lie between A_1 and B_1; $AM=\frac{a}{\cos\alpha}$, $BM=\frac{b}{\cos\beta}$, $A_1M=a\tan\alpha$, $B_1M=b\tan\beta$. The duration of motion of the ray is $\frac{a}{v_1\cos\alpha}+\frac{b}{v_2\cos\beta}$. The problem reduces to finding the minimum of the function $f(\alpha, \beta)=\frac{a}{v_1\cos\alpha}+\frac{b}{v_2\cos\beta}$ provided that $a\tan\alpha+b\tan\beta=c$. **2051.** $\alpha=\beta$. **2052.** $I_1:I_2:I_3=\frac{1}{R_1}:\frac{1}{R_2}:\frac{1}{R_3}$. **Hint.** Find the minimum of the function $f(I_1, I_2, I_3)=I_1^2R_1+I_2^2R_2+I_3^2R_3$ provided that $I_1+I_2+I_3=I$. **2053.** The isolated point (0, 0). **2054.** Cusp of second kind (0, 0). **2055.** Tacnode (0, 0). **2056.** Isolated point (0, 0). **2057.** Node (0, 0). **2058.** Cusp of first kind (0, 0). **2059.** Node (0, 0). **2060.** Node (0, 0). **2061.** Origin is isolated point if $a>b$; it is a cusp of the first kind if $a=b$, and a node if $a<b$. **2062.** If among the quantities a, b, and c, none are equal, then the curve does not have any singular points. If $a=b<c$, then $A(a, 0)$ is an isolated point; if $a<b=c$, then $B(b, 0)$ is a node; if $a=b=c$, then $A(a, 0)$ is a cusp of the first kind. **2063.** $y=\pm x$. **2064.** $y^2=2px$. **2065.** $y=\pm R$. **2066.** $x^{2/3}+y^{2/3}=l^{2/3}$. **2067.** $xy=\frac{1}{2}S$. **2068.** A pair of conjugate equilateral hyperbolas, whose equations, if the axes of symmetry of the ellipses are taken as the coordinate axes, have the form $xy=\pm\frac{S}{2\pi}$. **2069.** a) The discriminant curve $y=0$ is the locus of points of inflection and of the envelope of the given family; b) the discriminant curve $y=0$ is the locus of cusps and of the envelope of the family; c) the discriminant curve $y=0$ is the locus of cusps and is not an envelope; d) the discriminant curve decomposes into the straight lines: $x=0$ (locus of nodes) and $x=a$ (envelope). **2070.** $y=\frac{v_0^2}{2g}-\frac{gx^2}{2v_0^2}$. **2071.** $7\frac{1}{3}$. **2072.** $\sqrt{9+4\pi^2}$. **2073.** $\sqrt{3}(e^t-1)$. **2074.** 42. **2075.** 5. **2076.** x_0+z_0. **2077.** $11+\frac{\ln 10}{9}$

2079. a) Straight line; b) parabola; c) ellipse; d) hyperbola. **2080.** 1) $\frac{da}{dt}a^0$ 2) $a\frac{da^0}{dt}$; 3) $\frac{da}{dt}a^0+a\frac{da^0}{dt}$. **2081.** $\frac{d}{dt}(abc)=\left(\frac{da}{dt}bc\right)+\left(a\frac{db}{dt}c\right)+\left(ab\frac{dc}{dt}\right)$ **2082.** $4t(t^2+1)$. **2083.** $x=3\cos t$; $y=4\sin t$ (ellipse); for $t=0$, $\mathbf{v}=4\mathbf{j}$, $\mathbf{w}=-3\mathbf{i}$; fo $t=\frac{\pi}{4}$, $\mathbf{v}=-\frac{3\sqrt{2}}{2}\mathbf{i}+2\sqrt{2}\mathbf{j}$, $w=-\frac{3\sqrt{2}}{2}\mathbf{i}-2\sqrt{2}\mathbf{j}$; for $t=\frac{\pi}{2}$, $\mathbf{v}=-3\mathbf{i}$, $\mathbf{w}=-4\mathbf{j}$. **2084.** $x=2\cos t$, $y=2\sin t$, $z=3t$ (screw-line); $\mathbf{v}=-2\mathbf{i}\sin t+2\mathbf{j}\cos t+3\mathbf{k}$; $v=\sqrt{13}$ for any t; $\mathbf{w}=-2\mathbf{i}\cos t-2\mathbf{j}\sin t$; $w=2$ for any t for $t=0$, $\mathbf{v}=2\mathbf{j}+3\mathbf{k}$, $\mathbf{w}=-2\mathbf{i}$; for $t=\frac{\pi}{2}$, $\mathbf{v}=-2\mathbf{i}+3\mathbf{k}$, $\mathbf{w}=-2\mathbf{j}$

2085. $x=\cos\alpha\cos\omega t$; $y=\sin\alpha\cos\omega t$; $z=\sin\omega t$ (circle); $\mathbf{v}=-\omega\mathbf{i}\cos\alpha\sin\omega t-\omega\mathbf{j}\sin\alpha\sin\omega t+\omega\mathbf{k}\cos\omega t$; $v=|\omega|$; $\mathbf{w}=-\omega^2\mathbf{i}\cos\alpha\cos\omega t-\omega^2\mathbf{j}\sin\alpha\cos\omega t-\omega^2\mathbf{k}\sin\omega t$; $w=\omega^2$. 2086. $v=\sqrt{v_{x_0}^2+v_{y_0}^2+(v_{x_0}-gt)^2}$; $w_x=w_y=0$; $w_z=-g$; $w=g$. 2088. $\omega\sqrt{a^2+h^2}$, where $\omega=\frac{d\theta}{dt}$ is the angular speed of rotation of the screw. 2089. $\sqrt{a^2\omega^2+v_0^2-2a\omega v_0\sin\omega t}$. 2090. $\boldsymbol{\tau}=\frac{\sqrt{2}}{2}(\mathbf{i}+\mathbf{k})$; $\boldsymbol{\nu}=-\mathbf{j}$; $\boldsymbol{\beta}=\frac{\sqrt{2}}{2}(\mathbf{i}-\mathbf{k})$. 2091. $\boldsymbol{\tau}=\frac{1}{\sqrt{3}}[(\cos t-\sin t)\mathbf{i}+(\sin t+\cos t)\mathbf{j}+\mathbf{k}]$; $\boldsymbol{\nu}=-\frac{1}{\sqrt{2}}[(\sin t+\cos t)\mathbf{i}+(\sin t-\cos t)\mathbf{j}]$; $\cos(\widehat{\boldsymbol{\tau}, z})=\frac{\sqrt{3}}{3}$; $\cos(\widehat{\boldsymbol{\nu}, z})=0$. 2092. $\boldsymbol{\tau}=\frac{\mathbf{i}+4\mathbf{j}+2\mathbf{k}}{\sqrt{21}}$; $\boldsymbol{\nu}=\frac{-4\mathbf{i}+5\mathbf{j}-8\mathbf{k}}{\sqrt{105}}$; $\boldsymbol{\beta}=\frac{-2\mathbf{i}+\mathbf{k}}{\sqrt{5}}$. 2093. $\frac{x-a\cos t}{-a\sin t}=\frac{y-a\sin t}{a\cos t}=\frac{z-bt}{b}$ (tangent); $\frac{x-a\cos t}{b\sin t}=\frac{y-a\sin t}{-b\cos t}=\frac{z-bt}{a}$ (binormal); $\frac{x-a\cos t}{\cos t}=\frac{y-a\sin t}{\sin t}=\frac{z-bt}{0}$ (principal normal). The direction cosines of the tangent are $\cos\alpha=-\frac{a\sin t}{\sqrt{a^2+b^2}}$; $\cos\beta=\frac{a\cos t}{\sqrt{a^2+b^2}}$; $\cos\gamma=\frac{b}{\sqrt{a^2+b^2}}$. The direction cosines of the principal normal are $\cos\alpha_1=\cos t$; $\cos\beta_1=\sin t$; $\cos\gamma_1=0$. 2094. $2x-z=0$ (normal plane); $y-1=0$ (osculating plane); $x+2z-5=0$ (rectifying plane). 2095. $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-8}{12}$ (tangent); $x+4y+12z-114=0$ (normal plane); $12x-6y+z-8=0$ (osculating plane). 2096. $\frac{x-\frac{t^2}{4}}{t^2}=\frac{y-\frac{t^3}{3}}{t}=\frac{z-\frac{t^2}{2}}{1}$ (tangent); $\frac{x-\frac{t^4}{4}}{t^3+2t}=\frac{y-\frac{t^3}{3}}{1-t^4}=\frac{z-\frac{t^2}{2}}{-2t^3-t}$ (principal normal); $\frac{x-\frac{t^4}{4}}{1}=\frac{y-\frac{t^3}{3}}{-2t}=\frac{z-\frac{t^2}{2}}{t^2}$ (binormal); $M_1\left(\frac{1}{4}, -\frac{1}{3}, \frac{1}{2}\right)$; $M_2\left(4, -\frac{8}{3}, 2\right)$. 2097. $\frac{x-2}{1}=\frac{y+2}{-1}=\frac{z-2}{2}$ (tangent); $x+y=0$ (osculating plane); $\frac{x-2}{1}=\frac{y+2}{-1}=\frac{z-2}{-1}$ (principal normal); $\frac{x-2}{+1}=\frac{y+2}{1}=\frac{z-2}{0}$ (binormal); $\cos\alpha_2=\frac{1}{\sqrt{2}}$; $\cos\beta_2=\frac{1}{\sqrt{2}}$, $\cos\gamma_2=0$. 2098. a) $\frac{x-\frac{R}{2}}{2}=\frac{y-\frac{R}{2}}{0}=\frac{z-\frac{\sqrt{2}}{2}R}{-\sqrt{2}}$ (tangent); $x\sqrt{2}-z=0$ (normal plane); b) $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-2}{4}$ (tangent); $x+y+4z-10=0$ (normal plane); c) $\frac{x-2}{2\sqrt{3}}=\frac{y-2\sqrt{3}}{1}=\frac{z-3}{-2\sqrt{3}}$ (tangent); $2\sqrt{3}\,x+y-2\sqrt{3}z=0$ (normal plane); 2099. $x+y=0$. 2100. $x-y-z\sqrt{2}=0$. 2101. a) $4x-y-z-9=0$; b) $9x-6y+2z-18=0$; c) $b^2x_0^3x-a^2y_0^3y+(a^2-b^2)z_0^3z=a^2b^2(a^2-b^2)$. 2102. $6x-8y-z+3=0$ (osculating plane); $\frac{x-1}{31}=\frac{y-1}{26}=\frac{z-1}{-22}$ (principal normal); $\frac{x-1}{-6}=\frac{y-1}{8}=\frac{z-1}{1}$

(binormal). 2103. $bx-z=0$ (osculating plane); $\left.\begin{matrix}x=0,\\ z=0\end{matrix}\right\}$ (principal normal); $\left.\begin{matrix}x+bz=0,\\ y=0\end{matrix}\right\}$ (binormal); $\tau=\frac{i+bk}{\sqrt{1+b^2}}$; $\beta=\frac{-bi+k}{\sqrt{1+b^2}}$; $\nu=j$. 2106. $2x+3y+19z-27=0$. 2107. a) $\sqrt{2}$; b) $\frac{\sqrt{6}}{4}$. 2108. a) $K=\frac{e^{-t}\sqrt{2}}{3}$; $T=\frac{e^{-t}}{3}$; b) $K=T=\frac{1}{2a\cosh^2 t}$. 2109. a) $R=\varrho=\frac{(y+a)^2}{a}$; b) $R=\varrho=\frac{(p^4+2x^4)^3}{8p^4x^3}$. 2111. $\frac{av^2}{a^2+b^2}$. 2112. When $t=0$, $K=2$, $w_\tau=0$, $w_n=2$; when $t=1$, $K=\frac{1}{7}\sqrt{\frac{19}{14}}$, $w_\tau=\frac{22}{\sqrt{14}}$, $w_n=2\sqrt{\frac{19}{14}}$.

Chapter VII

2113. $4\frac{2}{3}$. 2114. $\ln\frac{25}{24}$ 2115. $\frac{\pi}{12}$. 2116. $\frac{9}{4}$. 2117. 50.4. 2118. $\frac{\pi a^2}{2}$. 2119. 2.4. 2120. $\frac{\pi}{6}$. 2121. $x=\frac{y^2}{4}-1$; $x=2-y$; $y=-6$; $y=2$. 2122. $y=x^2$; $y=x+9$; $x=1$; $x=3$. 2123. $y=x$; $y=10-x$; $y=0$; $y=4$. 2124. $y=\frac{x}{3}$; $y=2x$; $x=1$; $x=3$. 2125. $y=0$; $y=\sqrt{25-x^2}$; $x=0$; $x=3$. 2126. $y=x^2$; $y=x+2$; $x=-1$; $x=2$. 2127. $\int_0^1 dy\int_0^2 f(x,\ y)\,dx=\int_0^2 dx\int_0^1 f(x,\ y)\,dy$.

2128. $\int_0^1 dy\int_y^1 f(x,\ y)\,dx=\int_0^1 dx\int_0^x f(x,\ y)\,dy$. 2129. $\int_0^1 dy\int_0^{2-y} f(x,\ y)\,dx=$ $=\int_0^1 dx\int_0^1 f(x,\ y)\,dy+\int_1^2 dx\int_0^{2-x} f(x,\ y)\,dy$. 2130. $\int_1^2 dx\int_{2x}^{2x+3} f(x,\ y)\,dy=$ $=\int_2^4 dy\int_1^{\frac{y}{2}} f(x,\ y)\,dx+\int_4^5 dy\int_1^2 f(x,\ y)\,dx+\int_5^7 dy\int_{\frac{y-3}{2}}^2 f(x,\ y)\,dx$.

2131. $\int_0^1 dy\int_{-y}^{y} f(x,\ y)\,dx+\int_1^{\sqrt{2}} dy\int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} f(x,\ y)\,dx=\int_{-1}^0 dx\int_{-x}^{\sqrt{2-x^2}} f(x,y)\,dy+$ $+\int_0^1 dx\int_x^{\sqrt{2-x^2}} f(x,\ y)\,dy$. 2132. $\int_{-1}^1 dx\int_{2x^2}^2 f(x,\ y)\,dy=\int_0^2 dy\int_{-\sqrt{\frac{y}{2}}}^{\sqrt{\frac{y}{2}}} f(x,\ y)\,dx$.

2133. $$\int_{-2}^{-1} dx \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x, y)\, dy + \int_{-1}^{1} dx \int_{-\sqrt{4-x^2}}^{-\sqrt{1-x^2}} f(x, y)\, dy + \int_{-1}^{1} dx \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} f(x, y)\, dy + \int_{1}^{2} dx \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x, y)\, dy = \int_{-2}^{-1} dy \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} f(x, y)\, dx + \int_{-1}^{1} dy \int_{-\sqrt{4-y^2}}^{-\sqrt{1-y^2}} f(x, y)\, dx + \int_{-1}^{1} dy \int_{\sqrt{1-y^2}}^{\sqrt{4-y^2}} f(x, y)\, dx + \int_{1}^{2} dy \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} f(x, y)\, dx.$$

2134. $$\int_{-3}^{-2} dx \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} f(x, y)\, dy + \int_{-2}^{2} dx \int_{-\sqrt{1+x^2}}^{\sqrt{1+x^2}} f(x, y)\, dy + \int_{2}^{3} dx \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} f(x, y)\, dy = \int_{-\sqrt{5}}^{-1} dy \int_{-\sqrt{9-y^2}}^{-\sqrt{y^2-1}} f(x, y)\, dx + \int_{-\sqrt{5}}^{-1} dy \int_{\sqrt{y^2-1}}^{\sqrt{9-y^2}} f(x, y)\, dx + \int_{-1}^{1} dy \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} f(x, y)\, dx + \int_{1}^{\sqrt{5}} dy \int_{-\sqrt{9-y^2}}^{-\sqrt{y^2-1}} f(x, y)\, dy + \int_{1}^{\sqrt{5}} dy \int_{\sqrt{y^2-1}}^{\sqrt{9-y^2}} f(x, y)\, dx.$$

2135. a) $\int_{0}^{1} dx \int_{0}^{1-x} f(x, y)\, dy = \int_{0}^{1} dy \int_{0}^{1-y} f(x, y)\, dx$; b) $\int_{-a}^{a} dx \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} f(x, y)\, dy = \int_{-a}^{a} dy \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} f(x, y)\, dx$; c) $\int_{0}^{1} dx \int_{-\sqrt{x-x^2}}^{\sqrt{x-x^2}} f(x, y)\, dy = \int_{-1/2}^{1/2} dy \int_{\frac{1-\sqrt{1-4y^2}}{2}}^{\frac{1+\sqrt{1-4y^2}}{2}} f(x, y)\, dx$; d) $\int_{-1}^{1} dx \int_{x}^{1} f(x, y)\, dy = \int_{-1}^{1} dy \int_{-1}^{y} f(x, y)\, dx$; e) $\int_{0}^{a} dy \int_{y}^{y+2a} f(x, y)\, dx = \int_{0}^{a} dx \int_{0}^{x} f(x, y)\, dy + \int_{a}^{2a} dx \int_{0}^{a} f(x, y)\, dy + \int_{2a}^{3a} dx \int_{x-2a}^{a} f(x, y)\, dy$.

2136. $\int_{0}^{48} dy \int_{\frac{y}{12}}^{\sqrt{\frac{y}{3}}} f(x, y)\, dx$. **2137.** $\int_{0}^{2} dy \int_{\frac{y}{3}}^{\frac{y}{2}} f(x, y)\, dx + \int_{2}^{3} dy \int_{\frac{y}{3}}^{1} f(x, y)\, dx$.

2138. $\int_{0}^{\frac{a}{2}} dy \int_{\sqrt{a^2-2ay}}^{\sqrt{a^2-y^2}} f(x, y)\, dx + \int_{\frac{a}{2}}^{a} dy \int_{0}^{\sqrt{a^2-y^2}} f(x, y)\, dx$.

(binormal). 2103. $bx-z=0$ (osculating plane); $\left.\begin{array}{r}x=0,\\ z=0\end{array}\right\}$ (principal normal); $\left.\begin{array}{r}x+bz=0,\\ y=0\end{array}\right\}$ (binormal); $\tau=\frac{i+bk}{\sqrt{1+b^2}}$; $\beta=\frac{-bi+k}{\sqrt{1+b^2}}$; $\nu=j$. 2106. $2x+3y+19z-27=0$. 2107. a) $\sqrt{2}$; b) $\frac{\sqrt{6}}{4}$. 2108. a) $K=\frac{e^{-t}\sqrt{2}}{3}$; $T=\frac{e^{-t}}{3}$; b) $K=T=\frac{1}{2a\cosh^2 t}$. 2109. a) $R=\varrho=\frac{(y+a)^2}{a}$; b) $R=\varrho=\frac{(p^4+2x^4)^3}{8p^4x^3}$. 2111. $\frac{av^2}{a^2+b^2}$. 2112. When $t=0$, $K=2$, $w_\tau=0$, $w_n=2$; when $t=1$, $K=\frac{1}{7}\sqrt{\frac{19}{14}}$, $w_\tau=\frac{22}{\sqrt{14}}$, $w_n=2\sqrt{\frac{19}{14}}$.

Chapter VII

2113. $4\frac{2}{3}$. 2114. $\ln\frac{25}{24}$. 2115. $\frac{\pi}{12}$ 2116. $\frac{9}{4}$. 2117. 50.4. 2118. $\frac{\pi a^2}{2}$. 2119. 2.4. 2120. $\frac{\pi}{6}$. 2121. $x=\frac{y^2}{4}-1$; $x=2-y$; $y=-6$; $y=2$. 2122. $y=x^2$; $y=x+9$; $x=1$; $x=3$. 2123. $y=x$; $y=10-x$; $y=0$; $y=4$. 2124. $y=\frac{x}{3}$; $y=2x$; $x=1$; $x=3$. 2125. $y=0$; $y=\sqrt{25-x^2}$; $x=0$; $x=3$. 2126. $y=x^2$; $y=x+2$; $x=-1$; $x=2$. 2127. $\int_0^1 dy\int_0^2 f(x, y)\,dx=\int_0^2 dx\int_0^1 f(x, y)\,dy$.

2128. $\int_0^1 dy\int_y^1 f(x, y)\,dx=\int_0^1 dx\int_0^x f(x, y)\,dy$. 2129. $\int_0^1 dy\int_0^{2-y} f(x, y)\,dx=\int_0^1 dx\int_0^1 f(x, y)\,dy+\int_1^2 dx\int_0^{2-x} f(x, y)\,dy$. 2130. $\int_1^2 dx\int_{2x}^{2x+3} f(x, y)\,dy=\int_2^4 dy\int_1^{\frac{y}{2}} f(x, y)\,dx+\int_4^5 dy\int_1^2 f(x, y)\,dx+\int_5^7 dy\int_{\frac{y-3}{2}}^2 f(x, y)\,dx$.

2131. $\int_0^1 dy\int_{-y}^{y} f(x, y)\,dx+\int_1^{\sqrt{2}} dy\int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} f(x, y)\,dx=\int_{-1}^0 dx\int_{-x}^{\sqrt{2-x^2}} f(x, y)\,dy+\int_0^1 dx\int_x^{\sqrt{2-x^2}} f(x, y)\,dy$. 2132. $\int_{-1}^1 dx\int_{2x^2}^2 f(x, y)\,dy=\int_0^2 dy\int_{-\sqrt{\frac{y}{2}}}^{\sqrt{\frac{y}{2}}} f(x, y)\,dx$.

2133. $\int\limits_{-2}^{-1} dx \int\limits_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x, y)\,dy + \int\limits_{-1}^{1} dx \int\limits_{-\sqrt{4-x^2}}^{-\sqrt{1-x^2}} f(x, y)\,dy + \int\limits_{-1}^{1} dx \int\limits_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} f(x, y)\,dy + \int\limits_{1}^{2} dx \int\limits_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x, y)\,dy = \int\limits_{-2}^{-1} dy \int\limits_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} f(x, y)\,dx + \int\limits_{-1}^{1} dy \int\limits_{-\sqrt{4-y^2}}^{-\sqrt{1-y^2}} f(x, y)\,dx + \int\limits_{-1}^{1} dy \int\limits_{\sqrt{1-y^2}}^{\sqrt{4-y^2}} f(x, y)\,dx + \int\limits_{1}^{2} dy \int\limits_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} f(x, y)\,dx.$

2134. $\int\limits_{-3}^{-2} dx \int\limits_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} f(x, y)\,dy + \int\limits_{-2}^{2} dx \int\limits_{-\sqrt{1+x^2}}^{\sqrt{1+x^2}} f(x, y)\,dy + \int\limits_{2}^{3} dx \int\limits_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} f(x, y)\,dy = \int\limits_{-\sqrt{5}}^{-1} dy \int\limits_{-\sqrt{9-y^2}}^{-\sqrt{y^2-1}} f(x, y)\,dx + \int\limits_{-\sqrt{5}}^{-1} dy \int\limits_{\sqrt{y^2-1}}^{\sqrt{9-y^2}} f(x, y)\,dx + \int\limits_{-1}^{1} dy \int\limits_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} f(x, y)\,dx + \int\limits_{1}^{\sqrt{5}} dy \int\limits_{-\sqrt{9-y^2}}^{-\sqrt{y^2-1}} f(x, y)\,dy + \int\limits_{1}^{\sqrt{5}} dy \int\limits_{\sqrt{y^2-1}}^{\sqrt{9-y^2}} f(x, y)\,dx.$ **2135.** a) $\int\limits_{0}^{1} dx \int\limits_{0}^{1-x} f(x, y)\,dy = \int\limits_{0}^{1} dy \int\limits_{0}^{1-y} f(x, y)\,dx;$ b) $\int\limits_{-a}^{a} dx \int\limits_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} f(x, y)\,dy = \int\limits_{-a}^{a} dy \int\limits_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} f(x, y)\,dx;$ c) $\int\limits_{0}^{1} dx \int\limits_{-\sqrt{x-x^2}}^{\sqrt{x-x^2}} f(x, y)\,dy = \int\limits_{-1/2}^{1/2} dy \int\limits_{\frac{1-\sqrt{1-4y^2}}{2}}^{\frac{1+\sqrt{1-4y^2}}{2}} f(x, y)\,dx;$ d) $\int\limits_{-1}^{1} dx \int\limits_{x}^{1} f(x, y)\,dy = \int\limits_{-1}^{1} dy \int\limits_{-1}^{y} f(x, y)\,dx;$ e) $\int\limits_{0}^{a} dy \int\limits_{y}^{y+2a} f(x, y)\,dx = \int\limits_{0}^{a} dx \int\limits_{0}^{x} f(x, y)\,dy + \int\limits_{a}^{2a} dx \int\limits_{0}^{a} f(x, y)\,dy + \int\limits_{2a}^{3a} dx \int\limits_{x-2a}^{a} f(x, y)\,dy.$

2136. $\int\limits_{0}^{48} dy \int\limits_{\frac{y}{12}}^{\sqrt{\frac{y}{3}}} f(x, y)\,dx.$ **2137.** $\int\limits_{0}^{2} dy \int\limits_{\frac{y}{3}}^{\frac{y}{2}} f(x, y)\,dx + \int\limits_{2}^{3} dy \int\limits_{\frac{y}{3}}^{1} f(x, y)\,dx.$

2138. $\int\limits_{0}^{\frac{a}{2}} dy \int\limits_{\sqrt{a^2-2ay}}^{\sqrt{a^2-y^2}} f(x, y)\,dx + \int\limits_{\frac{a}{2}}^{a} dy \int\limits_{0}^{\sqrt{a^2-y^2}} f(x, y)\,dx.$

2139. $\int_0^{\frac{a\sqrt{3}}{2}} dy \int_{\frac{a}{2}}^{a} f(x, y)\,dx + \int_{\frac{a\sqrt{3}}{2}}^{a} dy \int_{a-\sqrt{a^2-y^2}}^{a} f(x, y)\,dx.$

2140. $\int_0^a dy \int_{\frac{y^2}{4a}}^{a-\sqrt{a^2-y^2}} f(x, y)\,dx + \int_0^a dy \int_{a+\sqrt{a^2-y^2}}^{2a} f(x, y)\,dx + \int_0^{2\sqrt{2a}} dy \int_{\frac{y^2}{4a}}^{2a} f(x, y)\,dx.$

2141. $\int_{-1}^{0} dx \int_0^{\sqrt{1-x^2}} f(x, y)\,dy + \int_0^1 dx \int_0^{1-x} f(x, y)\,dy.$ **2142.** $\int_0^{\frac{1}{2}} dx \int_0^{\sqrt{2x}} f(x, y)\,dy +$ $+ \int_{\frac{1}{2}}^{\sqrt{2}} dx \int_0^1 f(x,y)\,dy + \int_{\sqrt{2}}^{\sqrt{3}} dx \int_0^{\sqrt{3-x^2}} f(x, y)\,dy.$ **2143.** $\int_0^{\frac{R\sqrt{2}}{2}} dy \int_y^{\sqrt{R^2-y^2}} f(x, y)\,dx.$

2144. $\int_0^1 dy \int_{\arcsin y}^{\pi-\arcsin y} f(x, y)\,dx.$ **2145.** $\frac{1}{6}$. **2146.** $\frac{1}{6}$. **2147.** $\frac{\pi}{2}a$. **2148.** $\frac{\pi}{6}$.

2149. 6. **2150.** $\frac{1}{2}$. **2151.** $\ln 2$ **2152.** a) $\frac{4}{3}$; b) $\frac{15\pi-16}{150}$; c) $2\frac{2}{5}$.

2153. $\frac{8\sqrt{2}}{21}p^5$. **2154.** $\int_1^3 dx \int_0^{\sqrt{1-(x-2)^2}} xy\,dy = \frac{4}{3}$. **2155.** $\frac{8}{3}a\sqrt{2a}$.

2156. $\frac{5}{2}\pi R^3$. Hint. $\iint\limits_{(S)} y\,dx\,dy = \int_0^{2\pi R} dx \int_0^{y=f(x)} y\,dy =$ $= \int_0^{2\pi} R(1-\cos t)\,dt \int_0^{R(1-\cos t)} y\,dy$, where the last integral is obtained from the preceding one by the substitution $x = R(t-\sin t)$. **2157.** $\frac{R^4}{80}$. **2158.** $\frac{1}{6}$.

2159. $a^2 + \frac{R^2}{2}$. **2160.** $\int_0^{\frac{\pi}{4}} d\varphi \int_0^{\frac{1}{\cos\varphi}} rf(r\cos\varphi, r\sin\varphi)\,dr +$ $+ \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int_0^{\frac{1}{\sin\varphi}} rf(r\cos\varphi, r\sin\varphi)\,dr.$ **2161.** $\int_0^{\frac{\pi}{4}} d\varphi \int_0^{\frac{2}{\cos\varphi}} rf(r^2)\,dr.$

2162. $\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\varphi \int\limits_{0}^{\frac{1}{\sin\varphi}} rf(r\cos\varphi,\ r\sin\varphi)\,dr$. 2163. $\int\limits_{0}^{\frac{\pi}{4}} f(\tan\varphi)\,d\varphi \int\limits_{0}^{\frac{\sin\varphi}{\cos^2\varphi}} r\,dr +$

$+\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} f(\tan\varphi)\,d\varphi \int\limits_{0}^{\frac{1}{\sin\varphi}} r\,dr + \int\limits_{\frac{3\pi}{4}}^{\pi} f(\tan\varphi)\,d\varphi \int\limits_{0}^{\frac{\sin\varphi}{\cos^2\varphi}} r\,dr.$

2164. $\int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\varphi \int\limits_{0}^{a\sqrt{\cos 2\varphi}} rf(r\cos\varphi,\ r\sin\varphi)\,dr + \int\limits_{\frac{3\pi}{4}}^{5\pi} d\varphi \int\limits_{0}^{a\sqrt{\cos 2\varphi}} rf(r\cos\varphi,\ r\sin\varphi)\,dr.$

2165. $\int\limits_{0}^{\frac{\pi}{2}} d\varphi \int\limits_{0}^{a\cos\varphi} r^2\sin\varphi\,dr = \frac{a^3}{12}$. 2166. $\frac{3}{2}\pi a^4$. 2167. $\frac{\pi a^3}{3}$.

2168. $\left(\frac{22}{9}+\frac{\pi}{2}\right)a^3$. 2169. $\frac{\pi a^3}{6}$. 2170. $\left(\frac{\pi}{3}-\frac{16\sqrt{2}-20}{9}\right)\frac{a^3}{2}$.

2171. $\frac{2}{3}\pi ab$. **Hint.** The Jacobian is $I=abr$. The limits of integration are $0\leqslant\varphi\leqslant 2\pi$, $0\leqslant r\leqslant 1$. 2172. $\int\limits_{\frac{\alpha}{1+\alpha}}^{\frac{\beta}{1+\beta}} dv \int\limits_{0}^{\frac{c}{1-v}} f(u-uv,\ uv)\,u\,du$. **Solution.** We have $x=u(1-v)$ and $y=uv$; the Jacobian is $I=u$. We define the limits u as functions of v: when $x=0$, $u(1-v)=0$, whence $u=0$ (since $1-v\neq 0$); when $x=c$, $u=\frac{c}{1-v}$. Limits of variation of v: since $y=\alpha x$, it follows that $uv=\alpha u(1-v)$, whence $v=\frac{\alpha}{1+\alpha}$; for $y=\beta x$ we find $v=\frac{\beta}{1+\beta}$. 2173. $I=\frac{1}{2}\left[\int\limits_{0}^{1} du \int\limits_{-u}^{u} f\left(\frac{u+v}{2},\ \frac{u-v}{2}\right)dv +\right.$

$\left.+\int\limits_{1}^{2} du \int\limits_{u-2}^{2-u} f\left(\frac{u+v}{2},\ \frac{u-v}{2}\right)dv\right]=\frac{1}{2}\left[\int\limits_{-1}^{0} dv \int\limits_{-v}^{2+v} f\left(\frac{u+v}{2},\ \frac{u-v}{2}\right)du+\right.$

$\left.+\int\limits_{0}^{1} dv \int\limits_{v}^{2-v} f\left(\frac{u+v}{2},\ \frac{u-v}{2}\right)du\right]$. **Hint.** After change of variables, the equations of the sides of the square will be $u=v$; $u+v=2$; $u-v=2$; $u=-v$.

2174. $ab\left[\left(\frac{a^2}{h^2}-\frac{b^2}{k^2}\right)\arctan\frac{ak}{bh}+\frac{ab}{hk}\right]$. **Solution.** The equation of the curve

$r^4=r^2\left(\frac{a^2}{h^2}\cos^2\varphi-\frac{b^2}{k^2}\sin^2\varphi\right)$, whence the lower limit for r will be 0 and the upper limit, $r=\sqrt{\frac{a^2}{h^2}\cos^2\varphi-\frac{b^2}{k^2}\sin^2\varphi}$. Since r must be real, it follows that $\frac{a^2}{h^2}\cos^2\varphi-\frac{b^2}{k^2}\sin^2\varphi\geqslant 0$; whence for the first quadrantal angle we have $\tan\varphi\leqslant\frac{ak}{bh}$. Due to symmetry of the region of integration relative to the axes, we can compute $\frac{1}{4}$ of the entire integral, confining ourselves to the first quadrant: $\iint\limits_{(S)} dx\,dy=4\int\limits_0^{\arctan\frac{ak}{bh}} d\varphi\int\limits_0^{\sqrt{\frac{a^2}{h^2}\cos^2\varphi-\frac{b^2}{k^2}\sin^2\varphi}} abr\,dr$.

2175. a) $4\frac{1}{2}$; $\int\limits_0^1 dy\int\limits_{-\sqrt{y}}^{\sqrt{y}} dx+\int\limits_1^2 dy\int\limits_{y-2}^{\sqrt{y}} dx$; b) $\frac{\pi a^2}{4}-\frac{a^2}{2}$; $\int\limits_0^a dx\int\limits_{a-x}^{\sqrt{a^2-x^2}} dy$.

2176. a) $\frac{9}{2}$; b) $\left(2+\frac{\pi}{4}\right)a^2$. **2177.** $\frac{7a^2}{120}$. **2178.** $\frac{10}{3}a^2$. **2179.** π **Hint.** $-1\leqslant x\leqslant 1$. **2180.** $\frac{16}{3}\sqrt{15}$. **2181.** $3\left(\frac{\pi}{4}+\frac{1}{2}\right)$. **2182.** $\frac{4\pi}{3}-\sqrt{3}$.

2183. $\frac{5}{4}\pi a^2$. **2184.** 6. **2185.** 10π. **Hint.** Change the variables $x-2y=u$, $3x+4y=v$. **2186.** $\frac{1}{3}(b-a)(\beta-\alpha)$. **2187.** $\frac{1}{3}(\beta-\alpha)\ln\frac{b}{a}$.

2188. $v=\int\limits_0^1 dy\int\limits_y^1(1-x)\,dx=\int\limits_0^1 dx\int\limits_0^x(1-x)\,dy$. **2193.** $\frac{\pi a^3}{6}$. **2194.** $\frac{3}{4}$. **2195.** $\frac{1}{6}$.

2196. $\frac{a^3}{3}$. **2197.** $\frac{\pi r^4}{4a}$. **2198.** $\frac{48\sqrt{6}}{5}$. **2199.** $\frac{88}{105}$. **2200.** $\frac{a^3}{18}$. **2201.** $\frac{abc}{3}$.

2202. $\pi a^3(\alpha-\beta)$. **2203.** $\frac{4}{3}\pi a^3(2\sqrt{2}-1)$. **2204.** $\frac{4}{3}\pi a^3(\sqrt{2}-1)$.

2205. $\frac{\pi a^3}{3}$. **2206.** $\frac{4}{3}\pi abc$. **2207.** $\frac{\pi a^3}{3}(6\sqrt{3}-5)$. **2208.** $\frac{32}{9}a^3$.

2209. $\pi a(1-e^{-R^2})$. **2210.** $\frac{3\pi ab}{2}$. **2211.** $\frac{3\sqrt{3}-2}{2}$. **2212.** $\frac{\sqrt{2}}{2}(2\sqrt{2}-1)$. **Hint.** Change the variables $xy=u$, $\frac{y}{x}=v$. **2213.** $\frac{1}{2}\sqrt{a^2b^2+b^2c^2+c^2a^2}$

2214. $4(m-n)R^2$. **2215.** $\frac{\sqrt{2}}{2}a^2$. **Hint.** Integrate in the yz-plane. **2216.** $4a^2$.

2217. $8a^2\arcsin\frac{b}{a}$. **2218.** $\frac{1}{3}\pi a^2(3\sqrt{3}-1)$. **2219.** $8a^2$. **2220.** $3\pi a^2$. **Hint.** Pass to polar coordinates. **2221.** $\sigma=\frac{2}{3}\pi a^2\left[\left(1+\frac{R^2}{a^2}\right)^{\frac{3}{2}}-1\right]$. **Hint.** Pass to polar coordinates. 2222. $\frac{16}{9}a^3$ and $8a^2$. Hint. Pass to polar coordinates.

2223. $8a^2 \arctan\frac{\sqrt{2}}{5}$ Hint. $\sigma=\int_0^{\frac{a}{2}} dx\int_0^{\frac{a}{2}} \frac{a\,dy}{\sqrt{a^2-x^2-y^2}}=8a\int_0^{\frac{a}{2}} \arcsin\frac{a}{2\sqrt{a^2-x^2}}dx.$

Integrate by parts, and then change the variable $x=\frac{a\sqrt{3}}{2}\sin t$; transform the answer. 2224 $\frac{\pi}{4}\left(b\sqrt{b^2+c^2}-a\sqrt{a^2+c^2}+c^2\ln\frac{b+\sqrt{b^2+c^2}}{a+\sqrt{a^2+c^2}}\right)$. Hint. Pass to polar coordinates 2225. $\frac{2\pi\delta R^2}{3}$. 2226. $\frac{a^3b}{12}$; $\frac{a^2b^2}{24}$. 2227. $\bar{x}=\frac{12-\pi^2}{3(4-\pi)}$; $\bar{y}=\frac{\pi}{6(4-\pi)}$. 2228. $\bar{x}=\frac{5}{6}a$; $\bar{y}=0$. 2229. $\bar{x}=\frac{2a\sin\alpha}{3\alpha}$; $\bar{y}=0$. 2230. $\bar{x}=\frac{2}{5}$; $\bar{y}=0$. 2231. $I_X=4$ 2232. a) $I_0=\frac{\pi}{32}(D^4-d^4)$; b) $I_X=\frac{\pi}{64}(D^4-d^4)$.

2233. $I=\frac{2}{3}a^4$. 2234. $\frac{8}{5}a^4$. Hint. $I=\int_0^a dx\int_{-\sqrt{ax}}^{\sqrt{ax}}(y+a)^2\,dy.$

2235. $16\ln 2-9\frac{3}{8}$. Hint. The distance of the point (x, y) from the straight line $x=y$ is equal to $d=\frac{x-y}{\sqrt{2}}$ and is found by means of the normal equation of the straight line. 2236. $I=\frac{1}{40}ka^5[7\sqrt{2}+3\ln(\sqrt{2}+1)]$, where k is the proportionality factor. Hint. Placing the coordinate origin at the vertex, the distance from which is proportional to the density of the lamina, we direct the coordinate axes along the sides of the square. The moment of inertia is determined relative to the x-axis Passing to polar coordinates, we have

$$I_x=\int_0^{\frac{\pi}{4}} d\varphi\int_0^{a\sec\varphi} kr(r\sin\varphi)^2 r\,dr+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi\int_0^{a\operatorname{cosec}\varphi} kr(r\sin\varphi)^2 r\,dr$$

2237. $I_0=\frac{35}{16}\pi a^4$.

2238. $I_0=\frac{\pi a^4}{2}$. 2239. $\frac{35}{12}\pi a^4$. Hint. For the variables of integration take t and y (see Problem 2156). 2240. $\int_0^1 dx\int_0^{1-x} dy\int_0^{1-x-y} f(x, y, z)\,dz$

2241. $\int_{-R}^{R} dx\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} dy\int_0^H f(x, y, z)\,dz.$

2242. $\int_{-a}^{a} dx\int_{-\frac{b}{a}\sqrt{a^2-x^2}}^{\frac{b}{a}\sqrt{a^2-x^2}} dy\int_{c\sqrt{\frac{x^2}{a^2}+\frac{y^2}{b^2}}}^{c} f(x, y, z)\,dz.$

2243. $\int\limits_{-1}^{1} dx \int\limits_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy \int\limits_{0}^{\sqrt{1-x^2-y^2}} f(x, y, z)\,dz$.

2244. $\frac{8}{15}(31+12\sqrt{2}-27\sqrt{3})$. 2245. $\frac{4\pi\sqrt{2}}{3}$. 2246. $\frac{\pi^2a^2}{8}$. 2247. $\frac{1}{720}$. 2248. $\frac{1}{2}\ln 2-\frac{5}{16}$. 2249. $\frac{\pi a^5}{5}\left(18\sqrt{3}-\frac{97}{6}\right)$. 2250. $\frac{59}{480}\pi R^5$. 2251. $\frac{\pi abc^2}{4}$. 2252. $\frac{4}{5}\pi abc$. 2253. $\frac{\pi h^2R^2}{4}$. 2254. πR^3. 2255. $\frac{8}{9}a^2$. 2256. $\frac{8}{3}r^3\left(\pi-\frac{4}{3}\right)$. 2257. $\frac{4}{15}\pi R^5$. 2258. $\frac{\pi}{10}$. 2259. $\frac{32}{9}a^2h$. 2260. $\frac{3}{4}\pi a^3$. Solution. $v=$

$$=2\int\limits_0^{2a} dx \int\limits_0^{\sqrt{2ax-x^2}} dy \int\limits_0^{\frac{x^2+y^2}{2a}} dz=2\int\limits_0^{\frac{\pi}{2}} d\varphi \int\limits_0^{2a\cos\varphi} r\,dr \int\limits_0^{\frac{r^2}{2a}} dh=$$

$$=2\int\limits_0^{\frac{\pi}{2}} d\varphi \int\limits_0^{2a\cos\varphi} \frac{r^3dr}{2a}=\frac{1}{a}\int\limits_0^{\frac{\pi}{2}} \frac{(2a\cos\varphi)^4}{4}\,d\varphi=\frac{3}{4}\pi a^3.$$

2261. $\frac{2\pi a^3\sqrt{2}}{3}$. **Hint.** Pass to spherical coordinates. 2262. $\frac{19}{6}\pi$. **Hint.** Pass to cylindrical coordinates. 2263. $\frac{a^3}{9}(3\pi-4)$. 2264. πabc. 2265. $\frac{abc}{2}(a+b+c)$. 2266. $\frac{ab}{24}(6c^2-a^2-b^2)$.

2267. $\bar{x}=0$; $\bar{y}=0$; $\bar{z}=\frac{2}{5}a$. **Hint.** Introduce spherical coordinates.

2268. $\bar{x}=\frac{4}{3}$, $\bar{y}=0$, $\bar{z}=0$. 2269. $\frac{\pi a^2h}{12}(3a^2+4h^2)$. **Hint.** For the axis of the cylinder we take the z-axis, for the plane of the base of the cylinder, the xy-plane The moment of inertia is computed about the x-axis. After passing to cylindrical coordinates, the square of the distance of an element $r\,d\varphi\,dr\,dz$ from the x-axis is equal to $r^2\sin^2\varphi+z^2$. 2270. $\frac{\pi\varrho ha^2}{60}(2h^2+3a^2)$. **Hint.** The base of the cone is taken for the xy-plane, the axis of the cone, for the z-axis. The moment of inertia is computed about the x-axis. Passing to cylindrical coordinates, we have for points of the surface of the cone: $r=\frac{a}{h}(h-z)$; and the square of the distance of the element $r\,d\varphi\,dr\,dz$ from the x-axis is equal to $r^2\sin^2\varphi+z^2$. 2271. $2\pi k\varrho h(1-\cos\alpha)$, where k is the proportionality factor and ϱ is the density. **Solution.** The vertex of the cone is taken for the coordinate origin and its axis is the z-axis. If we introduce spherical coordinates, the equation of the lateral surface of the cone will be $\psi=\frac{\pi}{2}-\alpha$, and the equation of the plane of the base will be $r=\frac{h}{\sin\psi}$. From the symmetry it follows that the resulting stress is directed along the z-axis. The mass of an element of volume $dm=\varrho r^2\cos\psi\,d\varphi\,d\psi\,dr$, where ϱ is the density. The component of attraction, along the z-axis, by this element of unit mass lying at the point 0 is equal to $\frac{k\,dm}{r^2}\sin\psi=k\varrho\sin\psi\cos\psi\,d\psi\,d\varphi\,dr$.

The resulting attraction is equal to $\int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}-\alpha} d\psi \int_0^{h\,\mathrm{cosec}\,\psi} k\varrho \sin\psi\cos\psi\, dr.$

2272. Solution. We introduce cylindrical coordinates (ϱ, φ, z) with origin at the centre of the sphere and with the z-axis passing through a material point whose mass we assume equal to m. We denote by ξ the distance of this point from the centre of the sphere. Let $r=\sqrt{\varrho^2+(\xi-z)^2}$ be the distance from the element of volume dv to the mass m. The attractive force of the element of volume dv of the sphere and the material point m is directed along r and is numerically equal to $-k\gamma m\frac{dv}{r^2}$, where $\gamma=\frac{M}{\frac{4}{3}\pi R^3}$ is the density of the sphere and $dv=\varrho\, d\varphi\, d\varrho\, dz$ is the element of volume. The projection of this force on the z-axis is

$$dF=-\frac{km\gamma\, dv}{r^2}\cos(\widehat{rz})=-km\gamma\frac{\xi-z}{r^3}\varrho\, d\varphi\, d\varrho\, dz.$$

Whence

$$F=-km\gamma\int_0^{2\pi} d\varphi\int_{-R}^{R}(\xi-z)\,dz\int_0^{\sqrt{R^2-z^2}}\frac{\varrho\, d\varrho}{r^3}=km\gamma\frac{4}{3}\pi R^3\frac{1}{\xi^2}.$$

But since $\frac{4}{3}\gamma\pi R^3=M$, it follows that $F=\frac{kMm}{\xi^2}$. **2273.** $-\int_x^{\infty} y^2e^{-xy^2}\,dy-e^{-x^3}.$

2275. a) $\frac{1}{p}$ $(p>0)$; b) $\frac{1}{p-\alpha}$ for $p>\alpha$; c) $\frac{\beta}{p^2+\beta^2}$ $(p>0)$; d) $\frac{p}{p^2+\beta^2}$ $(p>0)$

2276. $-\frac{1}{n^2}$. **2277.** $\frac{2}{p^3}$. **Hint.** Differentiate $\int_0^{\infty} e^{-pt}\,dt=\frac{1}{p}$ twice. **2278.** $\ln\frac{\beta}{\alpha}$.

2279. $\arctan\frac{\beta}{m}-\arctan\frac{\alpha}{m}$. **2280.** $\frac{\pi}{2}\ln(1+\alpha)$. **2281.** $\pi(\sqrt{1-\alpha^2}-1)$.

2282. $\mathrm{arc\,cot}\frac{\alpha}{\beta}$. **2283.** 1. **2284.** $\frac{1}{2}$. **2285.** $\frac{\pi}{4}$. **2286.** $\frac{\pi}{4a^2}$. **Hint.** Pass to polar coordinates. **2287.** $\frac{\sqrt{\pi}}{2}$. **2288.** $\frac{\pi^2}{8}$. **2289.** Converges. **Solution.** Eliminate from S the coordinate origin together with its ε-neighbourhood, that is, consider $I_\varepsilon=\iint\limits_{(S_\varepsilon)}\ln\sqrt{x^2+y^2}\,dx\,dy$, where the eliminated region is a circle of radius ε with centre at the origin. Passing to polar coordinates, we have

$$I_\varepsilon=\int_0^{2\pi} d\varphi\int_\varepsilon^1 r\ln r\,dr=\int_0^{2\pi}\left[\frac{r^2}{2}\ln r\Big|_\varepsilon^1-\frac{1}{2}\int_\varepsilon^1 r\,dr\right]d\varphi=2\pi\left(\frac{\varepsilon^2}{4}-\frac{\varepsilon^2}{2}\ln\varepsilon-\frac{1}{4}\right).$$

Whence $\lim\limits_{\varepsilon\to 0} I_\varepsilon=-\frac{\pi}{2}$. **2290.** Converges for $\alpha>1$. **2291.** Converges. **Hint.** Surround the straight line $y=x$ with a narrow strip and put $\iint\limits_{(S)}\frac{dx\,dy}{\sqrt[3]{(x-y)^2}}=$

$= \lim_{\varepsilon \to 0} \int_0^1 dx \int_0^{x-\varepsilon} \frac{dy}{\sqrt[3]{(x-y)^2}} + \lim_{\delta \to 0} \int_0^1 dx \int_{x+\delta}^1 \frac{dy}{\sqrt[3]{(x-y)^2}}$. **2292.** Converges for $\alpha > \frac{3}{2}$. **2293.** 0. **2294.** $\ln \frac{\sqrt{5}+3}{2}$. **2295.** $\frac{ab(a^2+ab+b^2)}{3(a+b)}$. **2296.** $\frac{256}{15}a^3$. **2297.** $\frac{a^2}{3}\left[(1+4\pi^2)^{\frac{3}{2}}-1\right]$. **2298.** $\frac{a^5\sqrt{1+m^2}}{5m}$. **2299.** $a^2\sqrt{2}$. **2300.** $\frac{1}{54}(56\sqrt{7}-1)$. **2301.** $\frac{\sqrt{a^2+b^2}}{ab}\arctan\frac{2\pi b}{a}$. **2302.** $2\pi a^2$. **2303.** $\frac{16}{27}(10\sqrt{10}-1)$. **Hint.** $\int_C f(x, y)\,ds$ may be interpreted geometrically as the area of a cylindrical surface with generatrix parallel to the z-axis, with base, the contour of integration, and with altitudes equal to the values of the integrand. Therefore, $S = \int_C x\,ds$, where C is the arc OA of the parabola $y = \frac{3}{8}x^2$ that connects the points (0, 0) and (4,6). **2304.** $a\sqrt{3}$. **2305.** $2\left(b^2 + \frac{a^2 b}{\sqrt{a^2-b^2}}\arcsin\frac{\sqrt{a^2-b^2}}{a}\right)$. **2306.** $\sqrt{a^2+b^2}\left(\pi\sqrt{a^2+4\pi b^2} + \frac{a^2}{2b}\ln\frac{2\pi b+\sqrt{a^2+4\pi^2 b^2}}{a}\right)$. **2307.** $\left(\frac{4}{3}a, \frac{4}{3}a\right)$. **2308.** $2\pi a^2\sqrt{a^2+b^2}$. **2309.** $\frac{kMmb}{\sqrt{(a^2+b^2)^3}}$. **2310.** $40\frac{19}{30}$. **2311.** $-2\pi a^2$. **2312.** a) $\frac{4}{3}$; b) 0; c) $\frac{12}{5}$; d) -4; e) 4. **2313.** In all cases 4. **2314.** -2π. **Hint.** Use the parametric equations of a circle. **2315.** $\frac{4}{3}ab^2$. **2316.** $-2\sin 2$. **2317.** 0. **2318.** a) 8; b) 12; c) 2; d) $\frac{3}{2}$; e) $\ln(x+y)$; f) $\int_{x_1}^{x_2}\varphi(x)\,dx + \int_{y_1}^{y_2}\psi(y)\,dy$. **2319.** a) 62; b) 1; c) $\frac{1}{4}+\ln 2$; d) $1+\sqrt{2}$. **2320.** $\sqrt{1+a^2}-\sqrt{1+b^2}$. **2322.** a) $x^2+3xy-2y^2+C$; b) $x^3-x^2y+xy^2-y^3+C$; c) $e^{x-y}(x+y)+C$; d) $\ln|x+y|+C$. **2323.** $-2\pi a(a+b)$. **2324.** $-\pi R^2\cos^2\alpha$. **2325.** $\left(\frac{1}{6}+\frac{\pi\sqrt{2}}{16}\right)R^3$. **2326.** a) -20; b) $abc-1$; c) $5\sqrt{2}$; d) 0. **2327.** $I = \iint_{(S)} y^2\,dx\,dy$. **2328.** $-\frac{4}{3}$. **2329.** $\frac{\pi R^4}{2}$. **2330.** $-\frac{1}{3}$. **2331.** 0. **2332.** a) 0; b) $2n\pi$. **Hint** In Case (b), Green's formula is used in the region between the contour C and a circle of sufficiently small radius with centre at the coordinate origin **2333. Solution.** If we consider that the direction of the tangent coincides with that of positive circulation of the contour, then $\cos(X, n) = \cos(Y, t) = \frac{dy}{ds}$, hence, $\oint_C \cos(X, n)\,ds = \oint_C \frac{dy}{ds}\,ds = \oint_C dy = 0$ **2334.** $2S$, where S is the area bounded by the contour C. **2335.** -4. **Hint.** Green's formula is not applicable. **2336.** πab. **2337.** $\frac{3}{8}\pi a^2$. **2338.** $6\pi a^2$. **2339.** $\frac{3}{2}a^2$. **Hint.** Put

$y=tx$, where t is a parameter. **2340.** $\frac{a^2}{60}$. **2341.** $\pi(R+r)(R+2r)$; $6\pi R^2$ for $R=r$ **Hint.** The equation of an epicycloid is of the form $x=(R+r)\cos t-$ $-r\cos\frac{R+r}{r}t$, $y=(R+r)\sin t-r\sin\frac{R+r}{r}t$, where t is the angle of turn of the radius of a stationary circle drawn to the point of tangency. **2342.** $\pi(R-r)(R-2r)$, $\frac{3}{8}\pi R^2$ for $r=\frac{R}{4}$ **Hint.** The equation of the hypocycloid is obtained from the equation of the corresponding epicycloid (see Problem 2341) by replacing r by $-r$ **2343.** FR. **2344.** $mg(z_1-z_2)$. **2345.** $\frac{k}{2}(a^2-b^2)$, where k is a proportionality factor. **2346.** a) Potential, $U=mgz$, work, $mg(z_1-z_2)$; b) potential, $U=\frac{\mu}{r}$, work, $\frac{\mu}{\sqrt{a^2+b^2+c^2}}$; c) potential, $U=-\frac{k^2}{2}(x^2+y^2+z^2)$, work, $\frac{k^2}{2}(R^2-r^2)$. **2347.** $\frac{8}{3}\pi a^4$. **2348.** $\frac{2\pi a^2\sqrt{a^2+b^2}}{3}$. **2349.** 0. **2350.** $\frac{4}{3}\pi abc$. **2351.** $\frac{\pi a^4}{2}$. **2352.** $\frac{3}{4}$. **2353.** $\frac{25\sqrt{5}+1}{10(5\sqrt{5}-1)}a$. **2354.** $\frac{\pi\sqrt{2}}{2}h^4$. **2355.** a) 0; b) $-\iint\limits_{(S)}(\cos\alpha+\cos\beta+$ $+\cos\gamma)\,dS$. **2356.** 0. **2357.** 4π. **2358.** $-\pi a^2$. **2359.** $-a^3$. **2360.** $\frac{\partial R}{\partial y}=\frac{\partial Q}{\partial z}$, $\frac{\partial P}{\partial z}=\frac{\partial R}{\partial x}$, $\frac{\partial Q}{\partial x}=\frac{\partial P}{\partial y}$. **2361.** 0. **2362.** $2\iiint\limits_{(V)}(x+y+z)\,dx\,dy\,dz$.

2363. $2\iiint\limits_{(V)}\frac{dx\,dy\,dz}{\sqrt{x^2+y^2+z^2}}$. **2364.** $\iiint\limits_{(V)}\left(\frac{\partial^2U}{\partial x^2}+\frac{\partial^2U}{\partial y^2}+\frac{\partial^2U}{\partial z^2}\right)dx\,dy\,dz$.

2365. $3a^4$ **2366.** $\frac{a^3}{2}$. **2367.** $\frac{12}{5}\pi a^5$. **2368.** $\frac{\pi a^2b^2}{2}$ **2371.** Spheres; cylinders. **2372.** Cones. **2373.** Circles, $x^2+y^2=c_1^2$, $z=c_2$. **2376.** grad $U(A)=9\boldsymbol{i}-3\boldsymbol{j}-3\boldsymbol{k}$; $|\text{grad}\,U(A)|=\sqrt{99}=3\sqrt{11}$; $z^2=xy$; $x=y=z$. **2377.** a) $\frac{\boldsymbol{r}}{r}$; b) $2\boldsymbol{r}$. c) $-\frac{\boldsymbol{r}}{r^3}$; d) $f'(r)\frac{\boldsymbol{r}}{r}$ **2378.** grad $(\boldsymbol{cr})=\boldsymbol{c}$; the level surfaces are planes perpendicular to the vector $\boldsymbol{c}$. **2379.** $\frac{\partial U}{\partial r}=\frac{2U}{r}$, $\frac{\partial U}{\partial r}=|\text{grad}\,U|$ when $a=b=c$. **2380.** $\frac{\partial U}{\partial l}=$ $=-\frac{\cos(\boldsymbol{l},\boldsymbol{r})}{r^2}$; $\frac{\partial U}{\partial l}=0$ for $\boldsymbol{l}\perp\boldsymbol{r}$. **2382.** $\frac{2}{r}$. **2383.** div $\boldsymbol{a}=\frac{2}{r}f(r)+f'(r)$. **2385.** a) div $\boldsymbol{r}=3$, rot $\boldsymbol{r}=0$; b) div $(\boldsymbol{rc})=\frac{\boldsymbol{rc}}{r}$, rot $(\boldsymbol{rc})=\frac{\boldsymbol{r}\times\boldsymbol{c}}{r}$; c) div $(f(r)\,\boldsymbol{c})=$ $=\frac{f'(r)}{r}(\boldsymbol{c},\boldsymbol{r})$, rot $(f(r)\,\boldsymbol{c})=\frac{f'(r)}{r}\boldsymbol{c}\times\boldsymbol{r}$. **2386.** div $\boldsymbol{v}=0$; rot $\boldsymbol{v}=2\boldsymbol{\omega}$, where $\boldsymbol{\omega}=\omega\boldsymbol{k}$ **2387.** $2\omega\boldsymbol{n}^\circ$, where $\boldsymbol{n}^\circ$ is a unit vector parallel to the axis of rotation. **2388.** div grad $U=\frac{\partial^2U}{\partial x^2}+\frac{\partial^2U}{\partial y^2}+\frac{\partial^2U}{\partial z^2}$; rot grad $U=0$. **2391.** $3\pi R^2H$. **2392.** a) $\frac{1}{10}\pi R^2H(3R^2+2H^2)$; b) $\frac{3}{10}\pi R^2H(R^2+2H^2)$. **2393.** div $\boldsymbol{F}=0$ at all points except the origin. The flux is equal to $-4\pi m$. **Hint.** When calculating the flux, use the Ostrogradsky-Gauss theorem. **2394.** $2\pi^2h^2$. **2395.** $\frac{-\pi R^6}{8}$.

2396. $U=\int_{r_0}^{r} rf(r)\,dr$. **2397.** $\frac{m}{r}$. **2398.** a) No potential; b) $U=xyz+C$; c) $U=xy+xz+yz+C$. **2400.** Yes.

Chapter VIII

2401. $\frac{1}{2n-1}$. **2402.** $\frac{1}{2n}$. **2403.** $\frac{n}{2^{n-1}}$. **2404.** $\frac{1}{n^2}$. **2405.** $\frac{n+2}{(n+1)^2}$. **2406.** $\frac{2n}{3n+2}$. **2407.** $\frac{1}{n(n+1)}$. **2408.** $\frac{1\cdot3\cdot5\ldots(2n-1)}{1\cdot4\cdot7\ldots(3n-2)}$. **2409.** $(-1)^{n+1}$. **2410.** $n^{(-1)^{n+1}}$.
2416. Diverges. **2417.** Converges. **2418.** Diverges. **2419.** Diverges. **2420.** Diverges. **2421.** Diverges. **2422.** Diverges. **2423.** Diverges. **2424.** Diverges. **2425.** Converges. **2426.** Converges. **2427.** Converges. **2428.** Converges. **2429.** Converges. **2430.** Converges. **2431.** Converges. **2432.** Converges. **2433.** Converges. **2434.** Diverges. **2435.** Diverges. **2436.** Converges. **2437.** Diverges. **2438.** Converges. **2439.** Converges. **2440.** Converges. **2441.** Diverges. **2442.** Converges. **2443.** Converges. **2444.** Converges. **2445.** Converges. **2446.** Converges. **2447.** Converges. **2448.** Converges. **2449.** Converges. **2450.** Diverges. **2451.** Converges. **2452.** Diverges. **2453.** Converges. **2454.** Diverges. **2455.** Diverges. **2456.** Converges. **2457.** Diverges. **2458.** Converges. **2459.** Diverges. **2460.** Converges. **2461.** Diverges. **2462.** Converges. **2463.** Diverges. **2464.** Converges. **2465.** Converges. **2466.** Converges. **2467.** Diverges. **2468.** Diverges. **Hint.** $\frac{a_{n+1}}{a_n}>1$ **2470.** Converges conditionally. **2471.** Converges conditionally. **2472.** Converges absolutely **2473.** Diverges. **2474.** Converges conditionally. **2475.** Converges absolutely. **2476.** Converges conditionally. **2477.** Converges absolutely. **2478.** Converges absolutely. **2479.** Diverges. **2480.** Converges absolutely. **2481.** Converges conditionally. **2482.** Converges absolutely. **2484.** a) Diverges; b) converges absolutely; c) diverges; d) converges conditionally. **Hint.** In examples (a) and (d) consider the series $\sum_{k=1}^{\infty}(a_{2k-1}+a_{2k})$ and in examples (b) and (c) investigate separately the series $\sum_{k=1}^{\infty}a_{2k-1}$ and $\sum_{k=1}^{\infty}a_{2k}$. **2485.** Diverges. **2486.** Converges absolutely. **2487.** Converges absolutely. **2488.** Converges conditionally. **2489.** Diverges. **2490.** Converges absolutely. **2491.** Converges absolutely. **2492.** Converges absolutely. **2493.** Yes. **2494.** No. **2495.** $\sum_{n=1}^{\infty}\frac{1+(-1)^n}{3^n}$; converges. **2496.** $\sum_{n=1}^{\infty}\frac{1}{2n(2n-1)}$; converges. **2497.** Diverges. **2499.** Converges. **2500.** Converges.

2501. $|R_4|<\frac{1}{120}$, $|R_5|<\frac{1}{720}$; $R_4<0$, $R_5>0$. **2502.** $R_n<\frac{a_n}{2n+1}=\frac{1}{2^n(2n+1)\,n!}$
Hint. The remainder of the series may be evaluated by means of the sum of a geometric progression exceeding this remainder: $R_n=a_n\left[\frac{1}{2}\frac{1}{n+1}+\left(\frac{1}{2}\right)^2\frac{1}{(n+1)(n+2)}+\ldots\right]<a_n\left[\frac{1}{2}\cdot\frac{1}{n+1}+\left(\frac{1}{2}\right)^2\cdot\frac{1}{(n+1)^2}+\ldots\right]$.

2503. $R_n < \frac{n+2}{(n+1)(n+1)!}$; $R_{10} < 3\cdot 10^{-8}$. **2504.** $\frac{1}{n+1} < R_n < \frac{1}{n}$. Solution.

$$R_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \ldots > \frac{1}{(n+1)(n+2)} + \frac{1}{(n+2)(n+3)} + \ldots =$$
$$= \left(\frac{1}{n+1} - \frac{1}{n+2}\right) + \left(\frac{1}{n+2} - \frac{1}{n+3}\right) + \ldots = \frac{1}{n+1}, \; R_n < \frac{1}{n(n+1)} +$$
$$+ \frac{1}{(n+1)(n+2)} + \ldots = \frac{1}{n}.$$

2505. For the given series it is easy to find the exact value of the remainder:

$$R_n = \frac{1}{15}\left(n + \frac{16}{15}\right)\left(\frac{1}{4}\right)^{2n-2}.$$

Solution. $R_n = (n+1)\left(\frac{1}{4}\right)^{2n} + (n+2)\left(\frac{1}{4}\right)^{2n+2} + \ldots$

We multiply by $\left(\frac{1}{4}\right)^2$:

$$\frac{1}{16} R_n = (n+1)\left(\frac{1}{4}\right)^{2n+2} + (n+2)\left(\frac{1}{4}\right)^{2n+4} + \ldots$$

Whence we obtain

$$\frac{15}{16} R_n = n\left(\frac{1}{4}\right)^{2n} + \left(\frac{1}{4}\right)^{2n} + \left(\frac{1}{4}\right)^{2n+2} + \left(\frac{1}{4}\right)^{2n+4} + \ldots =$$
$$= n\left(\frac{1}{4}\right)^{2n} + \frac{\left(\frac{1}{4}\right)^{2n}}{1 - \frac{1}{16}} = \left(n + \frac{16}{15}\right)\left(\frac{1}{4}\right)^{2n}.$$

From this we find the above value of R_n. Putting $n=0$, we find the sum of the series $S = \left(\frac{16}{15}\right)^2$. **2506.** 99; 999. **2507.** 2; 3; 5. **2508.** $S=1$. **Hint.** $a_n = \frac{1}{n} - \frac{1}{n+1}$ **2509.** $S=1$ when $x>0$, $S=-1$ when $x<0$; $S=0$ when $x=0$. **2510.** Converges absolutely for $x>1$, diverges for $x \leqslant 1$. **2511.** Converges absolutely for $x>1$, converges conditionally for $0<x\leqslant 1$, diverges for $x \leqslant 0$. **2512.** Converges absolutely for $x>e$, converges conditionally for $1<x\leqslant e$, diverges for $x \leqslant 1$. **2513.** $-\infty < x < \infty$. **2514.** $-\infty < x < \infty$. **2515.** Converges absolutely for $x>0$, diverges for $x \leqslant 0$. **Solution.** 1) $|a_n| \leqslant \frac{1}{e^{nx}}$; and when $x>0$ the series with general term $\frac{1}{e^{nx}}$ converges; 2) $\frac{1}{e^{nx}} \geqslant 1$ for $x \leqslant 0$, and $\cos nx$ does not tend to zero as $n \to \infty$, since from $\cos nx \to 0$ it would follow that $\cos 2nx \to -1$; thus, the necessary condition for convergence is violated when $x \leqslant 0$. **2516.** Converges absolutely when $2k\pi < x < (2k+1)\pi$ $(k=0, \pm 1, \pm 2, \ldots)$; at the remaining points it diverges. **2517.** Diverges everywhere. **2518.** Converges absolutely for $x \neq 0$. **2519.** $x>1$, $x \leqslant -1$. **2520.** $x>3$, $x<1$. **2521.** $x \geqslant 1$, $x \leqslant -1$. **2522.** $x \geqslant 5\frac{1}{3}$, $x < 4\frac{2}{3}$. **2523.** $x>1$, $x<-1$. **2524.** $-1 < x < -\frac{1}{2}$, $\frac{1}{2} < x < 1$. **Hint.** For these values of x, both the series $\sum\limits_{k=1}^{\infty} x^k$ and the series $\sum\limits_{k=1}^{\infty} \frac{1}{2^k x^k}$ converge. When $|x| \geqslant 1$

and when $|x| \leqslant \frac{1}{2}$, the general term of the series does not tend to zero **2525.** $-1 < x < 0$, $0 < x < 1$. **2526.** $-1 < x < 1$. **2527.** $-2 \leqslant x < 2$. **2528.** $-1 < x < 1$ **2529.** $-\frac{1}{\sqrt{2}} \leqslant x \leqslant \frac{1}{\sqrt{2}}$. **2530.** $-1 < x \leqslant 1$. **2531.** $-1 < x < 1$ **2532.** $-1 < x < 1$. **2533.** $-\infty < x < \infty$. **2534.** $x = 0$. **2535.** $-\infty < x < \infty$. **2536.** $-4 < x < 4$. **2537.** $-\frac{1}{3} < x < \frac{1}{3}$. **2538.** $-2 < x < 2$. **2539.** $-e < x < e$.

2540. $-3 \leqslant x < 3$. **2541.** $-1 < x < 1$ **2542.** $-1 < x < 1$ **Solution.** The divergence of the series for $|x| \geqslant 1$ is obvious (it is interesting, however, to note that the divergence of the series at the end-points of the interval of convergence $x = \pm 1$ is detected not only with the aid of the necessary condition of convergence, but also by means of the d'Alembert test). When $|x| < 1$ we have

$$\lim_{n \to \infty} \left| \frac{(n+1)!\, x^{(n+1)!}}{n!\, x^{n!}} \right| = \lim_{n \to \infty} |(n+1)\, x^{n!\, n}| \leqslant \lim_{n \to \infty} (n+1)\, |x|^n = \lim_{n \to \infty} \frac{n+1}{\left|\frac{1}{x}\right|^n} = 0$$

(this equality is readily obtained by means of l'Hospital's rule).
2543. $-1 \leqslant x \leqslant 1$ **Hint.** Using the d'Alembert test, it is possible not only to find the interval of convergence, but also to investigate the convergence of the given series at the extremities of the interval of convergence. **2544.** $-1 \leqslant x \leqslant 1$. **Hint.** Using the Cauchy test, it is possible not only to find the interval of convergence, but also to investigate the convergence of the given series at the extremities of the interval of convergence. **2545.** $2 < x \leqslant 8$. **2546.** $-2 \leqslant x < 8$. **2547.** $-2 < x < 4$. **2548.** $1 \leqslant x \leqslant 3$ **2549.** $-4 \leqslant x \leqslant -2$. **2550.** $x = -3$ **2551.** $-7 < x < -3$ **2552.** $0 \leqslant x < 4$. **2553.** $-\frac{5}{4} < x < \frac{13}{4}$. **2554.** $-e-3 < x < e-3$. **2555.** $-2 \leqslant x \leqslant 0$. **2556.** $2 < x < 4$ **2557.** $1 < x \leqslant 3$. **2558.** $-3 \leqslant x \leqslant -1$ **2559.** $1 - \frac{1}{e} < x < 1 + \frac{1}{e}$ **Hint.** For $x = 1 \pm \frac{1}{e}$ the series diverges, since $\lim_{n \to \infty} \frac{\left(1 + \frac{1}{n}\right)^{n^2}}{e^n} = \frac{1}{\sqrt{e}} \neq 0$ **2560.** $-2 < x < 0$

2561. $1 < x \leqslant 3$ **2562.** $1 \leqslant x < 5$. **2563.** $2 \leqslant x \leqslant 4$. **2564.** $|z| < 1$ **2565.** $|z| < 1$ **2566.** $|z - 2i| < 3$ **2567.** $|z| < \sqrt{2}$ **2568.** $z = 0$ **2569.** $|z| < \infty$. **2570.** $|z| < \frac{1}{2}$ **2576.** $-\ln(1-x)$ $(-1 \leqslant x < 1)$ **2577.** $\ln(1+x)$ $(-1 < x \leqslant 1)$. **2578.** $\frac{1}{2} \ln \frac{1+x}{1-x}$ $(|x| < 1)$ **2579.** $\arctan x$ $(|x| \leqslant 1)$. **2580.** $\frac{1}{(x-1)^2}$ $(|x| < 1)$. **2581.** $\frac{1-x^2}{(1+x^2)^2}$ $(|x| < 1)$ **2582.** $\frac{2}{(1-x)^3}$ $(|x| < 1)$. **2583** $\frac{x}{(x-1)^2}$ $(|x| > 1)$. **2584.** $\frac{1}{2}\left(\arctan x - \frac{1}{2} \ln \frac{1-x}{1+x}\right)$ $(|x| < 1)$. **2585.** $\frac{\pi\sqrt{3}}{6}$. **Hint.** Consider the sum of the series $x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots$ (see Problem 2579) for $x = \frac{1}{\sqrt{3}}$.

2586. 3. **2587.** $a^x = 1 + \sum_{n=1}^{\infty} \frac{x^n \ln^n a}{n!}$, $-\infty < x < \infty$. **2588.** $\sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\left[1 + x - \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} - \ldots + (-1)^{\frac{n^2-n}{2}} \frac{x^n}{n!} + \ldots\right]$.

2589. $\cos(x+a)=\cos a - x\sin a - \frac{x^2}{2!}\cos a + \frac{x^3}{3!}\sin a + \frac{x^4}{4!}\cos a + \ldots$ $\ldots + \frac{x^n}{n!}\sin\left[a+\frac{(n+1)\pi}{2}\right] + \ldots, -\infty < x < \infty$. **2590.** $\sin^2 x = \frac{2x^2}{2!} - \frac{2^3x^4}{4!} + \frac{2^5x^6}{6!} - \ldots$ $\ldots + (-1)^{n-1}\frac{2^{2n-1}x^{2n}}{(2n)!} + \ldots, -\infty < x < \infty$. **2591** $\ln(2+x) = \ln 2 + \frac{x}{2} - \frac{x^2}{2\cdot 2^2} + \frac{x^3}{3\cdot 2^3} - \ldots + (-1)^{n-1}\frac{x^n}{n\cdot 2^n} + \ldots, -2 < x \leqslant 2$. **Hint.** When investigating the remainder, use the theorem on integrating a power series **2592.** $\frac{2x-3}{(x-1)^2} = -\sum_{n=0}^{\infty}(n+3)x^n, |x| < 1$. **2593.** $\frac{3x-5}{x^2-4x+3} = -\sum_{n=0}^{\infty}\left(1+\frac{2}{3^{n+1}}\right)x^n, |x| < 1$. **2594.** $xe^{-2x} = x + \sum_{n=2}^{\infty}\frac{(-1)^{n-1}2^{n-1}x^n}{(n-1)!}$, $-\infty < x < \infty$. **2595.** $e^{x^2} = 1 + \sum_{n=1}^{\infty}\frac{x^{2n}}{n!}, -\infty < x < \infty$ **2596.** $\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}$ $(-\infty < x < \infty)$ **2597.** $1 + \sum_{n=1}^{\infty}(-1)^n\frac{2^{n}x^{2n}}{(2n)!}$. **2598.** $1 + \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^n(2x)^{2n}}{(2n)!}$ $-\infty < x < \infty$. **2599.** $2\sum_{n=0}^{\infty}(-1)^n\frac{(n+2)3^{2n}\cdot x^{2n+1}}{(2n+1)!}$ $(-\infty < x < \infty)$.

2600. $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{9^{n+1}}$ $(-3 < x < 3)$. **2601.** $\frac{1}{2} + \frac{1}{2}\cdot\frac{x^2}{2^3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^4}{2^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^6}{2^7} + \ldots + \frac{1\cdot 3\cdot 5\ldots(2n-1)}{2\cdot 4\cdot 6\ldots 2n}\frac{x^{2n}}{2^{2n+1}} + \ldots$ $(-2 < x < 2)$

2602. $2\sum_{n=0}^{\infty}\frac{x^{2n+1}}{2n+1}$ $(|x| < 1)$ **2603.** $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}2^n-1}{n}x^n\left(-\frac{1}{2} < x \leqslant \frac{1}{2}\right)$.

2604 $x + \sum_{n=2}^{\infty}(-1)^n\frac{x^n}{(n-1)n}$ $(|x| \leqslant 1)$. **2605.** $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$ $(|x| \leqslant 1)$.

2606. $x + \frac{1}{2}\cdot\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \ldots + \frac{1\cdot 3\cdot 5\ldots(2n-1)}{2\cdot 4\cdot 6\ldots 2n}\frac{x^{2n+1}}{2n+1} + \ldots$ $(|x| \leqslant 1)$.

2607. $x - \frac{1}{2}\cdot\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} - \ldots + (-1)^n\frac{1\cdot 3\cdot 5\ldots(2n-1)}{2\cdot 4\cdot 6\ldots 2n}\frac{x^{2n+1}}{2n+1} + \ldots$ $(|x| \leqslant 1)$.

2608. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^{4n-3}x^{2n}}{(2n)!}$ $(-\infty < x < \infty)$. **2609.** $1 + \sum_{n=2}^{\infty}(-1)^{n-1}\frac{n-1}{n!}x^n$ $(-\infty < x < \infty)$. **2610.** $8 + 3\sum_{n=1}^{\infty}\frac{1+2^n+3^{n-1}}{n!}x^n$ $(-\infty < x < \infty)$.

2611. $2 + \frac{x}{2^2\cdot 3\cdot 1!} - \frac{2\cdot x^2}{2^5\cdot 3^2\cdot 2!} + \frac{2\cdot 5x^3}{2^8\cdot 3^3\cdot 3!} + \ldots + (-1)^{n-1}\frac{2\cdot 5\cdot 8\ldots(3n-4)x^n}{2^{3n-1}\cdot 3^n\cdot n!} + \ldots$

$(-\infty < x < \infty)$. **2612.** $\frac{1}{6} - \sum_{n=1}^{\infty} \left(\frac{1}{2^{n+1}} + \frac{1}{3^{n+1}} \right) x^n$ $(-2 < x < 2)$.

2613. $1 + \frac{3}{4} \sum_{n=1}^{\infty} \frac{(1+3^{2n-1})\, x^{2n}}{(2n)!}$ $(|x| < \infty)$. **2614.** $\sum_{n=0}^{\infty} \frac{x^{4n}}{4^{n+1}}$ $(|x| < \sqrt{2})$.

2615. $\ln 2 + \sum_{n=1}^{\infty} (-1)^{n-1} (1 + 2^{-n}) \frac{x^n}{n}$ $(-1 < x \leqslant 1)$. **2616.** $\sum_{n=0}^{\infty} (-1)^n \times$ $\times \frac{x^{2n+1}}{(2n+1)(2n+1)!}$ $(-\infty < x < \infty)$. **2617.** $x + \sum_{n=1}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)\, n!}$ $(|x| < \infty)$.

2618. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n^2}$ $(|x| \leqslant 1)$. **2619.** $x + \frac{1}{2 \cdot 5} x^5 + \frac{1 \cdot 3}{2^2 \cdot 9 \cdot 2!} x^9 + \ldots +$ $+ \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{2^n (4n+1)\, n!} x^{4n+1} + \ldots$ $(|x| < 1)$. **2620.** $x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots$

2621. $x - \frac{x^3}{3} + \frac{2x^5}{15} - \ldots$ **2622.** $e \left(1 - \frac{x^2}{2} + \frac{x^4}{6} - \ldots \right)$. **2623.** $1 + \frac{x^2}{2} + \frac{5x^4}{24} + \ldots$

2624. $-\left(\frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \ldots \right)$. **2625.** $x + x^2 + \frac{1}{3} x^3 + \ldots$ **2626.** Hint. Proceeding from the parametric equations of the ellipse $x = a \cos \varphi$, $y = b \sin \varphi$, compute the length of the ellipse and expand the expression obtained in a series of powers of ε. **2628.** $x^3 - 2x^2 - 5x - 2 = -78 + 59(x+4) - 14(x+4)^2 +$ $+ (x+4)^3$ $(-\infty < x < \infty)$. **2629.** $f(x+h) = 5x^3 - 4x^2 - 3x + 2 +$ $+ (15x^2 - 8x - 3)\, h + (15x - 4)\, h^2 + 5h^3$ $(-\infty < x < \infty;\ -\infty < h < \infty)$.

2630. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-1)^n}{n}$ $(0 < x \leqslant 2)$. **2631.** $\sum_{n=0}^{\infty} (-1)^n (x-1)^n$ $(0 < x < 2)$.

2632. $\sum_{n=0}^{\infty} (n+1)(x+1)^n$ $(-2 < x < 0)$. **2633.** $\sum_{n=0}^{\infty} (2^{-n-1} - 3^{-n-1})(x+4)^n$ $(-6 < x < -2)$. **2634.** $\sum_{n=0}^{\infty} (-1)^n \frac{(x+2)^{2n}}{3^{n+1}}$ $(-2 - \sqrt{3} < x < -2 + \sqrt{3})$.

2635. $e^{-2} \left[1 + \sum_{n=1}^{\infty} \frac{(x+2)^n}{n!} \right]$ $(|x| < \infty)$. **2636.** $2 + \frac{x-4}{2^2} - \frac{1}{4} \frac{(x-4)^2}{2^4} +$ $+ \frac{1 \cdot 3}{4 \cdot 6} \frac{(x-4)^3}{2^6} - \frac{1 \cdot 3 \cdot 5}{4 \cdot 6 \cdot 8} \frac{(x-4)^4}{2^8} + \ldots + (-1)^{n-1} \frac{1 \cdot 3 \cdot 5 \ldots (2n-3)}{4 \cdot 6 \cdot 8 \ldots 2n} \frac{(x-4)^n}{2^{2n}} + \ldots$ $(0 \leqslant x \leqslant 8)$. **2637.** $\sum_{n=1}^{\infty} (-1)^n \frac{\left(x - \frac{\pi}{2} \right)^{2n-1}}{(2n-1)!}$ $(|x| < \infty)$. **2638.** $\frac{1}{2} +$ $+ \sum_{n=1}^{\infty} (-1)^n \frac{4^{n-1} \left(x - \frac{\pi}{4} \right)^{2n-1}}{(2n-1)!}$ $(|x| < \infty)$. **2639.** $-2 \sum_{n=0}^{\infty} \frac{1}{2n+1} \left(\frac{1-x}{1+x} \right)^{2n+1}$ $(0 < x < \infty)$.

Hint. Make the substitution $\frac{1-x}{1+x}=t$ and expand $\ln x$ in powers of t. **2640.** $\frac{x}{1+x}+\frac{1}{2}\left(\frac{x}{1+x}\right)^2+\frac{1\cdot3}{2\cdot4}\left(\frac{x}{1+x}\right)^3+\ldots+\frac{1\cdot3\cdot5\ldots(2n-3)}{2\cdot4\cdot6\ldots(2n-2)}\left(\frac{x}{1+x}\right)^n+\ldots$ $\ldots\left(-\frac{1}{2}\leqslant x<\infty\right)$. **2641.** $|R|<\frac{e}{5!}<\frac{1}{40}$. **2642.** $|R|<\frac{1}{11}$. **2643.** $\frac{\pi}{6}\approx$ $\approx\frac{1}{2}+\frac{1}{2}\,\frac{\left(\frac{1}{2}\right)^3}{3}+\frac{1\cdot3}{2\cdot4}\,\frac{\left(\frac{1}{2}\right)^5}{5}\approx 0.523$. **Hint.** To prove that the error does not exceed 0.001, it is necessary to evaluate the remainder by means of a geometric progression that exceeds this remainder. **2644.** Two terms, that is, $1-\frac{x^2}{2}$. **2645.** Two terms, i.e., $x-\frac{x^3}{6}$. **2646.** Eight terms, i.e., $1+\sum_{n=1}^{7}\frac{1}{n!}$.

2647. 99; 999. **2648.** 1.92 **2649.** 4.8 $|R|<0.005$. **2650.** 2.087. **2651.** $|x|<0.69$; $|x|<0.39$; $|x|<0.22$. **2652.** $|x|<0.39$; $|x|<0.18$ **2653.** $\frac{1}{2}-\frac{1}{2^3\cdot3\cdot3!}\approx 0.4931$. **2654.** 0.7468. **2655.** 0.608 **2656.** 0.621 **2657.** 0.2505 **2658.** 0.026.

2659. $1+\sum_{n=1}^{\infty}(-1)^n\frac{(x-y)^{2n}}{(2n)!}$ $(-\infty<x<\infty;\ -\infty<y<\infty)$.

2660. $\sum_{n=1}^{\infty}(-1)^n\frac{(x-y)^{2n}-(x+y)^{2n}}{2\cdot(2n)!}$ $(-\infty<x<\infty;\ -\infty<y<\infty)$.

2661. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x^2+y^2)^{2n-1}}{(2n-1)!}$ $(-\infty<x<\infty;\ -\infty<y<\infty)$.

2662. $1+2\sum_{n=1}^{\infty}(y-x)^n$; $|x-y|<1$ **Hint.** $\frac{1-x+y}{1+x-y}=-1+\frac{2}{1-(y-x)}$. Use a geometric progression **2663.** $-\sum_{n=1}^{\infty}\frac{x^n+y^n}{n}$ $(-1\leqslant x<1;\quad -1\leqslant y<1)$.

Hint. $1-x-y+xy=(1-x)(1-y)$. **2664.** $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}+y^{2n+1}}{2n+1}$ $(-1\leqslant x\leqslant 1;$ $-1\leqslant y\leqslant 1)$. **Hint.** $\arctan\frac{x+y}{1-xy}=\arctan x+\arctan y$ (for $|x|\leqslant 1$, $|y|\leqslant 1$).

2665. $f(x+h,\ y+k)=ax^2+2bxy+cy^2+2(ax+by)h+2(bx+cy)k+ah^2+2bh+ck^2$. **2666.** $f(1+h,\ 2+k)-f(1,\ 2)=9h-21k+3h^2+3hk-12k^2+h^3-2k^3$. **2667.** $1+\sum_{n=1}^{\infty}\frac{[(x-2)+(y+2)]^n}{n!}$. **2668.** $1+\sum_{n=1}^{\infty}(-1)^n\frac{\left[x+\left(y-\frac{\pi}{2}\right)\right]^{2n}}{(2n)!}$.

2669. $1+x+\frac{x^2-y^2}{2!}+\frac{x^3-3xy^2}{3!}+\ldots$ **2670.** $1+x+xy+\frac{1}{2}x^2y+\ldots$

2671. $\frac{c_1+c_2}{2}-\frac{2(c_1-c_2)}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)x}{2n+1}$; $S(0)=\frac{c_1+c_2}{2}$; $S(\pm\pi)=\frac{c_1+c_2}{2}$.

2672. $\frac{b-a}{4}\pi-\frac{2(b-a)}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)x}{(2n+1)^2}+(a+b)\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sin nx}{n}$; $S(\pm\pi)=\frac{b-a}{2}\pi$. 2673. $\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}(-1)^n\frac{\cos nx}{n^2}$; $S(\pm\pi)=\pi^2$. 2674. $\frac{2}{\pi}\sinh a\pi\times$ $\times\left[\frac{1}{2a}+\sum_{n=1}^{\infty}\frac{(-1)^n}{a^2+n^2}(a\cos nx-n\sin nx)\right]$; $S(\pm\pi)=\cosh a\pi$. 2675. $\frac{2\sin a\pi}{\pi}\times$ $\times\sum_{n=1}^{\infty}(-1)^n\frac{n\sin nx}{a^2-n^2}$ if a is nonintegral; $\sin ax$ if a is an integer; $S(\pm\pi)=0$.

2676. $\frac{2\sin a\pi}{\pi}\left[\frac{1}{2a}+\sum_{n=1}^{\infty}(-1)^n\frac{a\cos nx}{a^2-n^2}\right]$ if a is nonintegral; $\cos ax$ if a is an integer; $S(\pm\pi)=\cos a\pi$. 2677. $\frac{2\sinh a\pi}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n\sin nx}{a^2+n^2}$; $S(\pm\pi)=0$.

2678. $\frac{2\sinh a\pi}{\pi}\left[\frac{1}{2a}+\sum_{n=1}^{\infty}(-1)^n\frac{a\cos nx}{a^2+n^2}\right]$; $S(\pm\pi)=\cosh a\pi$. 2679. $\sum_{n=1}^{\infty}\frac{\sin nx}{n}$.

2680. $\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{2n-1}$; a) $\frac{\pi}{4}$; b) $\frac{\pi}{3}$; c) $\frac{\pi}{2\sqrt{3}}$. 2681. a) $2\sum_{n=1}^{\infty}(-1)^{n-1}\times$ $\times\frac{\sin nx}{n}$; b) $\frac{\pi}{2}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2}$; $\frac{\pi^2}{8}$. 2682. a) $\sum_{n=1}^{\infty}b_n\sin nx$, where $b_{2k-1}=\frac{2\pi}{2k-1}-\frac{8}{\pi(2k-1)^3}$ and $b_{2k}=-\frac{\pi}{k}$; b) $\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}(-1)^n\frac{\cos nx}{n^2}$; 1) $\frac{\pi^2}{6}$. 2) $\frac{\pi^2}{12}$. 2683. a) $\frac{2}{\pi}\sum_{n=1}^{\infty}[1-(-1)^n e^{a\pi}]\frac{n\sin nx}{a^2+n^2}$; b) $\frac{e^{a\pi}-1}{a\pi}+$ $+\frac{2a}{\pi}\sum_{n=1}^{\infty}\frac{[(-1)^n e^{a\pi}-1]\cos nx}{a^2+n^2}$. 2684. a) $\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-\cos\frac{n\pi}{2}}{n}\sin nx$; b) $\frac{1}{2}+$ $+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin\frac{n\pi}{2}}{n}\cos nx$. 2685. a) $\frac{4}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sin(2n-1)x}{(2n-1)^2}$; b) $\frac{\pi}{4}-$ $-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos 2(2n-1)x}{(2n-1)^2}$ 2686. $\sum_{n=1}^{\infty}b_n\sin nx$, where $b_{2k}=(-1)^{k-1}\frac{1}{2k}$, $b_{2k+1}=$ $=(-1)^k\frac{2}{\pi(2k+1)^2}$ 2687. $\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{(2n-1)^3}$. 2688. $\frac{8}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n\sin nx}{4n^2-1}$.

2689. $\frac{2h}{\pi}\left(\frac{1}{2}+\sum_{n=1}^{\infty}\frac{\sin nh}{nh}\cos nx\right)$. **2690.** $\frac{2h}{\pi}\left[\frac{1}{2}+\sum_{n=1}^{\infty}\left(\frac{\sin nh}{nh}\right)^2\cos nx\right]$.

2691. $1-\frac{\cos x}{2}+2\sum_{n=2}^{\infty}(-1)^{n-1}\frac{\cos nx}{n^2-1}$. **2692.** $\frac{4}{\pi}\left[\frac{1}{2}+\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\cos 2nx}{4n^2-1}\right]$.

2694. **Solution.** 1) $a_{2n}=\frac{2}{\pi}\int_0^{\pi} f(x)\cos 2nx\,dx=\frac{2}{\pi}\int_0^{\frac{\pi}{2}} f(x)\cos 2nx\,dx+$ $+\frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi} f(x)\cos 2nx\,dx$. If we make the substitution $t=\frac{\pi}{2}-x$ in the first integral and $t=x-\frac{\pi}{2}$ in the second, then, taking advantage of the assumed identity $f\left(\frac{\pi}{2}+t\right)=-f\left(\frac{\pi}{2}-t\right)$, it will readily be seen that $a_{2n}=0$ $(n=0,\ 1,\ 2,\ \ldots)$;

2) $b_{2n}=\frac{2}{\pi}\int_0^{\pi} f(x)\sin 2nx\,dx=\frac{2}{\pi}\int_0^{\frac{\pi}{2}} f(x)\sin 2nx\,dx+\frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi} f(x)\sin 2nx\,dx.$

The same substitution as in Case (1), with account taken of the assumed identity $f\left(\frac{\pi}{2}+t\right)=f\left(\frac{\pi}{2}-t\right)$ leads to the equalities $b_{2n}=0$ $(n=1,\ 2,\ \ldots)$.

2695. $\frac{1}{2}-\frac{4}{\pi^2}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi x}{(2n+1)^2}$. **2696.** $1-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin 2n\pi x}{n}$.

2697. $\sinh l\left[\frac{1}{l}+2\sum_{n=1}^{\infty}(-1)^n\frac{l\cos\frac{n\pi x}{l}-\pi n\sin\frac{n\pi x}{l}}{l^2+n^2\pi^2}\right]$.

2698. $\frac{10}{\pi}\sum_{n=1}^{\infty}(-1)^n\frac{\sin\frac{n\pi x}{5}}{n}$ **2699.** a) $\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin 2(n-1)\pi x}{2n-1}$; b) 1 **2700** a) $\frac{2l}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin\frac{n\pi x}{l}}{n}$; b) $\frac{l}{2}-\frac{4l}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\frac{(2n-1)\pi x}{l}}{(2n-1)^2}$. **2701.** a) $\sum_{n=1}^{\infty}b_n\sin\frac{nx}{2}$, where $b_{2k+1}=\frac{8}{\pi}\left[\frac{\pi^2}{2k+1}-\frac{4}{(2k+1)^3}\right]$, $b_{2k}=-\frac{4\pi}{k}$; b) $\frac{4\pi^2}{3}-$ $-16\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\cos\frac{nx}{2}}{n^2}$. **2702.** a) $\frac{8}{\pi^2}\sum_{n=0}^{\infty}(-1)^n\frac{\sin\frac{(2n+1)\pi x}{2}}{(2n+1)^2}$, b) $\frac{1}{2}-$ $-\frac{4}{\pi^2}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi x}{(2n+1)^2}$. **2703.** $\frac{2}{3}-\frac{9}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos\frac{2n\pi x}{3}+\frac{1}{2\pi^2}\sum_{n=1}^{\infty}\frac{\cos 2n\pi x}{n^2}$.

Chapter IX

2704. Yes. **2705.** No. **2706.** Yes. **2707.** Yes. **2708.** Yes. **2709.** a) Yes; b) no. **2710.** Yes. **2714.** $y-xy'=0$. **2715.** $xy'-2y=0$. **2716.** $y-2xy'=0$. **2717.** $x\,dx+y\,dy=0$. **2718.** $y'=y$. **2719.** $3y^2-x^2=2xyy'$. **2720.** $xyy'(xy^2+1)=1$. **2721.** $y=xy'\ln\frac{x}{y}$. **2722.** $2xy''+y'=0$. **2723.** $y''-y'-2y=0$. **2724.** $y''+4y=0$. **2725.** $y''-2y'+y=0$. **2726.** $y''=0$. **2727.** $y'''=0$. **2728.** $(1+y'^2)y'''-3y'y''^2=0$. **2729.** $y^2-x^2=25$. **2730.** $y=xe^{2x}$. **2731.** $y=-\cos x$. **2732.** $y=\frac{1}{6}(-5e^{-x}+9e^x-4e^{2x})$. **2738.** 2.593 (exact value $y=e$). **2739.** 4.780 [exact value $y=3(e-1)$]. **2740.** 0.946 (exact value $y=1$). **2741.** 1.826 (exact value $y=\sqrt{3}$) **2742.** $\cot^2 y=\tan^2 x+C$. **2743.** $x=\frac{Cy}{\sqrt{1+y^2}}$; $y=0$. **2744.** $x^2+y^2=\ln Cx^2$. **2745.** $y=a+\frac{Cx}{1+ax}$. **2746.** $\tan y=C(1-e^x)^3$; $x=0$. **2747.** $y=C\sin x$. **2748.** $2e^{\frac{y^2}{2}}=\sqrt{e}\,(1+e^x)$. **2749.** $1+y^2=\frac{2}{1-x^2}$. **2750.** $y=1$. **2751.** $\operatorname{arc\,tan}(x+y)=x+C$. **2752.** $8x+2y+1=2\tan(4x+C)$. **2753.** $x+2y+3\ln|2x+3y-7|=C$. **2754.** $5x+10y+C=3\ln|10x-5y+6|$. **2755.** $\rho=\frac{C}{1-\cos\varphi}$ or $y^2=2Cx+C^2$. **2756.** $\ln\rho=\frac{1}{2\cos^2\varphi}-\ln|\cos\varphi|+C$ or $\ln|x|-\frac{y^2}{2x^2}=C$. **2757.** Straight line $y=Cx$ or hyperbola $y=\frac{C}{x}$. **Hint.** The segment of the tangent is equal to $\sqrt{y^2+\left(\frac{y}{y'}\right)^2}$. **2758.** $y^2-x^2=C$. **2759.** $y=Ce^{\frac{x}{a}}$. **2760.** $y^2=2px$. **2761.** $y=ax^2$. **Hint.** By hypothesis $\frac{\int_0^x xy\,dx}{\int_0^x y\,dx}=\frac{3}{4}x$. Differentiating twice with respect to x, we get a differential equation. **2762.** $y^2=\frac{1}{3}x$.

2763. $y=\sqrt{4-x^2}+2\ln\frac{2-\sqrt{4-x^2}}{x}$. **2764.** Pencil of lines $y=kx$. **2765.** Family of similar ellipses $2x^2+y^2=C^2$. **2766.** Family of hyperbolas $x^2-y^2=C$. **2767.** Family of circles $x^2+(y-b)^2=b^2$. **2768.** $y=x\ln\frac{C}{x}$. **2769.** $y=\frac{C}{x}-\frac{x}{2}$.

2770. $x=Ce^{\frac{x}{y}}$. **2771.** $(x-C)^2-y^2=C^2$; $(x-2)^2-y^2=4$; $y=\pm x$. **2772.** $\sqrt{\frac{x}{y}}+\ln|y|=C$. **2773.** $y=\frac{C}{2}x^2-\frac{1}{2C}$; $x=0$. **2774.** $(x^2+y^2)^3(x+y)^2C$. **2775.** $y=x\sqrt{1-\frac{3}{8}x}$. **2776.** $(x+y-1)^3=C(x-y+3)$. **2777.** $3x+y+2\times\ln|x+y-1|=C$. **2778.** $\ln|4x+8y+5|+8y-4x=C$. **2779.** $x^2=1-2y$.

2780. Paraboloid of revolution. **Solution.** By virtue of symmetry the sought-for mirror is a surface of revolution. The coordinate origin is located in the source of light; the x-axis is the direction of the pencil of rays. If a tangent at any point $M(x, y)$ of the curve, generated by the desired surface being cut by the xy-plane, forms with the x-axis an angle φ, and the segment connecting the origin with the point $M(x, y)$ forms an angle α, then $\tan\alpha = \tan 2\varphi = \frac{2\tan\varphi}{1-\tan^2\varphi}$. But $\tan\alpha = \frac{y}{x}$; $\tan\varphi = y'$. The desired differential equation is $y - yy'^2 = 2xy'$ and its solution is $y^2 = 2Cx + C^2$. The plane section is a parabola. The desired surface is a paraboloid of revolution. **2781.** $(x-y)^2 - Cy = 0$. **2782.** $x^2 = C(2y+C)$. **2783.** $(2y^2 - x^2)^3 = Cx^2$. **Hint.** Use the fact that the area is equal to $\int_a^x y\,dx$. **2784.** $y = Cx - x\ln|x|$. **2785.** $y = Cx + x^2$. **2786.** $y = \frac{1}{6}x^4 + \frac{C}{x^2}$. **2787.** $x\sqrt{1+y^2} + \cos y = C$. **Hint.** The equation is linear with respect to x and $\frac{dx}{dy}$. **2788.** $x = Cy^2 - \frac{1}{y}$. **2789.** $y = \frac{e^x}{x} + \frac{ab - e^a}{x}$. **2790.** $y = \frac{1}{2}(x\sqrt{1-x^2} + \arcsin x)\sqrt{\frac{1+x}{1-x}}$. **2791.** $y = \frac{x}{\cos x}$. **2792.** $y(x^2 + Cx) = 1$. **2793.** $y^2 = x\ln\frac{C}{x}$. **2794.** $x^2 = \frac{1}{y + Cy^2}$. **2795.** $y^3(3 + Ce^{\cos x}) = x$ **2797.** $xy = Cy^2 + a^2$. **2798.** $y^2 + x + ay = 0$. **2799.** $x = y\ln\frac{y}{a}$. **2800.** $\frac{a}{x} + \frac{b}{y} = 1$. **2801.** $x^2 + y^2 - Cy + a^2 = 0$. **2802.** $\frac{x^2}{2} + xy + y^2 = C$. **2803.** $\frac{x^3}{3} + xy^2 + x^2 = C$. **2804.** $\frac{x^4}{4} - \frac{3}{2}x^2y^2 + 2x + \frac{y^3}{3} = C$. **2805.** $x^2 + y^2 - 2\arctan\frac{y}{x} = C$. **2806.** $x^2 - y^2 = Cy^3$. **2807.** $\frac{x^2}{2} + ye^{\frac{x}{y}} = 2$ **2808.** $\ln|x| - \frac{y^2}{x} = C$. **2809.** $\frac{x}{y} + \frac{x^2}{2} = C$. **2810.** $\frac{1}{y}\ln x + \frac{1}{2}y^2 = C$. **2811.** $(x\sin y + y\cos y - \sin y)e^x = C$. **2812.** $(x^2C^2 + 1 - 2Cy)\times(x^2 + C^2 - 2Cy) = 0$; singular integral $x^2 - y^2 = 0$. **2813.** General integral $(y+C)^2 = x^3$; there is no singular integral. **2814.** General integral $\left(\frac{x^2}{2} - y + C\right)\times\left(x - \frac{y^2}{2} + C\right) = 0$; there is no singular integral. **2815.** General integral $y^2 + C^2 = 2Cx$; singular integral $x^2 - y^2 = 0$. **2816.** $y = \frac{1}{2}\cos x \pm \frac{\sqrt{3}}{2}\sin x$. **2817.** $\begin{cases} x = \sin p + \ln p, \\ y = p\sin p + \cos p + p + C. \end{cases}$ **2818.** $\begin{cases} x = e^p + pe^p + C, \\ y = p^2e^p. \end{cases}$ **2819.** $\begin{cases} x = 2p - \frac{2}{p} + C, \\ y = p^2 + 2\ln p. \end{cases}$ Singular solution: $y = 0$. **2820.** $4y = x^2 + p^2$, $\ln|p - x| = C + \frac{x}{p-x}$. **2821.** $\ln\sqrt{p^2 + y^2} + \arctan\frac{p}{y} = C$, $x = \ln\frac{y^2 + p^2}{2p}$. Singular solution: $y = e^x$.

2822. $y=\frac{1}{2}Cx^2+\frac{2}{C}$; $y=\pm 2x$. 2824. $\begin{cases} x=Ce^{-p}-2p+2, \\ y=C(1+p)e^{-p}-p^2+2. \end{cases}$

2823. $\begin{cases} x=\ln|p|-\arcsin p+C, \\ y=p+\sqrt{1-p^2}. \end{cases}$ 2825. $\begin{cases} x=\frac{1}{3}\left(Cp^{-\frac{1}{2}}-p\right), \\ y=\frac{1}{6}\left(2Cp^{\frac{1}{2}}+p^2\right). \end{cases}$ Hint. The differential equation from which x is defined as a function of p is homogeneous. 2826. $y=Cx+C^2$; $y=-\frac{x^2}{4}$. 2827. $y=Cx+C$; no singular solution 2828. $y=Cx+\sqrt{1+C^2}$; $x^2+y^2=1$. 2829. $y=Cx+\frac{1}{C}$; $y^2=4x$. 2830. $xy=C$ 2831. A circle and the family of its tangents. 2832. The astroid $x^{2/3}+y^{2/3}=a^{2/3}$. 2833. a) Homogeneous, $y=xu$; b) linear in x; $x=uv$; c) linear in y; $y=uv$; d) Bernoulli's equation; $y=uv$; e) with variables separable; f) Clairaut's equation; reduce to $y=xy'\pm\sqrt{y'^3}$; g) Lagrange's equation; differentiate with respect to x; h) Bernoulli's equation; $y=uv$; i) leads to equation with variables separable; $u=x+y$; j) Lagrange's equation; differentiate with respect to x; k) Bernoulli's equation in x; $x=uv$; l) exact differential equation; m) linear; $y=uv$; n) Bernoulli's equation; $y=uv$. 2834. a) $\sin\frac{y}{x}=-\ln|x|+C$; b) $x=y\cdot e^{Cy+1}$.

2835. $x^2+y^4=Cy^2$. 2836. $y=\frac{x}{x^2+C}$. 2837. $xy\left(C-\frac{1}{2}\ln^2 x\right)=1$. 2838. $y=Cx+C\ln C$; singular solution, $y=e^{-(x+1)}$. 2839. $y=Cx+\sqrt{-aC}$; singular solution, $y=\frac{a}{4x}$. 2840. $3y+\ln\frac{|x^2-1|}{(y+1)^6}=C$. 2841. $\frac{1}{2}e^{2x}-e^y-\arctan y-\frac{1}{2}\ln(1+y^2)=C$. 2842. $y=x^2\left(1+Ce^{\frac{1}{x}}\right)$. 2843. $x=y^2(C-e^{-y})$. 2844. $y=Ce^{-\sin x}+\sin x-1$. 2845. $y=ax+C\sqrt{1-x^2}$. 2846. $y=\frac{x}{x+1}(x+\ln|x|+C)$.

2847. $x=Ce^{\sin y}-2a(1+\sin y)$. 2848. $\frac{x^2}{2}+3x+y+\ln[(x-3)^{10}|y-1|^3]=C$.

2849. $2\arctan\frac{y-1}{2x}=\ln Cx$. 2850. $x^2=1-\frac{2}{y}+Ce^{-\frac{2}{y}}$. 2851. $x^3=Ce^y-y-2$

2852. $\sqrt{\frac{y}{x}}+\ln|x|=C$. 2853. $y=x\arcsin(Cx)$. 2854. $y^2=Ce^{-2x}+\frac{2}{5}\sin x+\frac{4}{5}\cos x$. 2855. $xy=C(y-1)$. 2856. $x=Ce^y-\frac{1}{2}(\sin y+\cos y)$. 2857. $py=C(p-1)$. 2858. $x^4=Ce^{4y}-y^3-\frac{3}{4}y^2-\frac{3}{8}y-\frac{3}{32}$ 2859. $(xy+C)(x^2y+C)=0$.

2860. $\sqrt{x^2+y^2}-\frac{x}{y}=C$. 2861. $xe^y-y^2=C$. 2862. $\begin{cases} x=\frac{C}{p^2}-\frac{\sqrt{1+p^2}}{2p}+\frac{1}{2p^2}\ln\left(p+\sqrt{1+p^2}\right), \\ y=2px+\sqrt{1+p^2}. \end{cases}$

2863. $y=xe^{Cx}$. 2864. $2e^x-y^4=Cy^2$. 2865. $\ln|y+2|+2\arctan\frac{y+2}{x-3}=C$. 2866.

$y^2+Ce^{-\frac{y^2}{2}}+\frac{1}{x}-2=0$. **2867.** $x^2\cdot y=Ce^{\frac{y}{a}}$. **2868.** $x+\frac{x}{y}=C$. **2869.** $y=\frac{C-x^4}{4(x^2-1)^{3/2}}$. **2870.** $y=C\sin x-a$. **2871.** $y=\frac{a^2\ln(x+\sqrt{a^2+x^2})+C}{x+\sqrt{a^2+x^2}}$. **2872.** $(y-Cx)\cdot(y^2-x^2+C)=0$. **2873.** $y=Cx+\frac{1}{C^2}$, $y=\frac{3}{2}\sqrt[3]{2x^2}$. **2874.** $x^3+x^2y-y^2x-y^3=C$. **2875.** $p^2+4y^2=Cy^3$. **2876.** $y=x-1$. **2877.** $y=x$. **2878.** $y=2$. **2879.** $y=0$. **2880.** $y=\frac{1}{2}(\sin x+\cos x)$. **2881.** $y=\frac{1}{4}(2x^2+2x+1)$. **2882.** $y=e^{-x}+2x-2$. **2883.** a) $y=x$; b) $y=Cx$, where C is arbitrary; the point (0,0) is a singular point of the differential equation. **2884.** a) $y^2=x$; b) $y^2=2px$; (0,0) is a singular point. **2885.** a) $(x-C)^2+y^2=C^2$; b) no solution; c) $x^2+y^2=x$; (0,0) is a singular point. **2886.** $y=e^{\frac{x}{y}}$. **2887.** $y=(\sqrt{2a}\pm\sqrt{x})^2$. **2888.** $y^2=1-e^{-x}$. **2889.** $r=Ce^{a\varphi}$. **Hint.** Pass to polar coordinates. **2890.** $3y^2-2x=0$. **2891.** $r=k\varphi$. **2892.** $x^2+(y-b)^2=b^2$. **2893.** $y^2+16x=0$. **2894.** Hyperbola $y^2-x^2=C$ or circle $x^2+y^2=C^2$. **2895.** $y=\frac{1}{2}(e^x+e^{-x})$. **Hint.** Use the fact that the area is equal to $\int_0^x y\,dx$ and the arc length, to $\int_0^x\sqrt{1+y'^2}\,dx$. **2896.** $x=\frac{a^2}{y}+Cy$. **2897.** $y^2=4C(C+a-x)$. **2898.** **Hint.** Use the fact that the resultant of the force of gravity and the centrifugal force is normal to the surface. Taking the y-axis as the axis of rotation and denoting by ω the angular velocity of rotation, we get for the plane axial cross-section of the desired surface the differential equation $g\frac{dy}{dx}=\omega^2x$. **2899.** $p=e^{-0.000167h}$. **Hint.** The pressure at each level of a vertical column of air may be considered as due solely to the pressure of the upper-lying layers. Use the law of Boyle-Mariotte, according to which the density is proportional to the pressure. The sought-for differential equation is $dp=-kp\,dh$. **2900.** $s=\frac{1}{2}kl\omega$. **Hint.** Equation $ds=k\omega\frac{l-x}{l}dx$. **2901.** $s=\left(p+\frac{1}{2}\omega\right)kl$. **2902.** $T=a+(T_0-a)e^{-kt}$. **2903.** In one hour. **2904.** $\omega=100\left(\frac{3}{5}\right)^t$ rpm. **2905.** 4.2% of the initial quantity Q_0 will decay in 100 years. **Hint.** Equation $\frac{dQ}{dt}=kQ$. $Q=Q_0\left(\frac{1}{2}\right)^{\frac{t}{1600}}$. **2906.** $t\approx 35.2$ sec. **Hint.** Equation $\pi(h^2-2h)\,dh=\pi\left(\frac{1}{10}\right)^2v\,dt$. **2907.** $\frac{1}{1024}$. **Hint.** $dQ=-kQ\,dh$. $Q=Q_0\left(\frac{1}{2}\right)^{\frac{h}{3}}$. **2908.** $v\to\sqrt{\frac{gm}{k}}$ as $t\to\infty$ (k is a proportionality factor). **Hint.** Equation $m\frac{dv}{dt}=mg-kv^2$; $v=\sqrt{\frac{gm}{k}}\tanh\left(t\sqrt{\frac{gk}{m}}\right)$. **2909.** 18.1 kg. **Hint.** Equation $\frac{dx}{dt}=k\left(\frac{1}{3}-\frac{x}{300}\right)$. **2910.** $i=\frac{E}{R^2+L^2\omega^2}[(R\sin\omega t-$

$-L\omega\cos\omega t)+L\omega e^{-\frac{R}{L}t}$]. **Hint.** Equation $Ri+L\frac{di}{dt}=E\sin\omega t$. **2911.** $y=x\ln|x|+C_1x+C_2$. **2912.** $1+C_1y^2=\left(C_2+\frac{C_1x}{\sqrt{2}}\right)^2$. **2913.** $y=\ln|e^{2x}+C_1|-x+C_2$. **2914.** $y=C_1+C_2\ln|x|$. **2915.** $y=C_1e^{C_2x}$ **2916.** $y=\pm\sqrt{C_1x+C_2}$. **2917.** $y=(1+C_1^2)\ln|x+C_1|-C_1x+C_2$. **2918.** $(x-C_1)=a\ln\left|\sin\frac{y-C_2}{a}\right|$. **2919.** $y=\frac{1}{2}(\ln|x|)^2+C_1\ln|x|+C_2$. **2920.** $x=\frac{1}{C_1}\ln\left|\frac{y}{y+C_1}\right|+C_2$; $y=C$. **2921.** $y=C_1e^{C_2x}+\frac{1}{C_2}$. **2922.** $y=\pm\frac{1}{2}\left[x\sqrt{C_1^2-x^2}+C_1^2\arcsin\frac{x}{C_1}\right]+C_2$ **2923.** $y=(C_1e^x+1)x+C_2$. **2924.** $y=(C_1x-C_1^2)e^{\frac{x}{C_1}+1}+C_2$; $y=\frac{e}{2}x^2+C$ (singular solution). **2925.** $y=C_1x(x-C_1)+C_2$; $y=\frac{x^3}{3}+C$ (singular solution). **2926.** $y=\frac{x^3}{12}+\frac{x^2}{2}+C_1x\ln|x|+C_2x+C_3$. **2927.** $y=\pm\sin(C_1\pm x)+C_2x+C_3$. **2928.** $y=x^3+3x$. **2929.** $y=\frac{1}{2}(x^2+1)$. **2930.** $y=x+1$. **2931.** $y=Cx^2$. **2932.** $y=C_1\times\frac{1+C_2e^x}{1-C_2e^x}$; $y=C$. **2933.** $x=C_1+\ln\left|\frac{y-C_2}{y+C_2}\right|$. **2934.** $x=C_1-\frac{1}{C_2}\ln\left|\frac{y}{y+C_2}\right|$. **2935.** $x=C_1y^2+y\ln y+C_2$. **2936.** $2y^2-4x^2=1$. **2937.** $y=x+1$. **2938.** $y=\frac{x^2-1}{2(e^2-1)}-\frac{e^2-1}{4}\ln|x|$ or $y=\frac{1-x^2}{2(e^2+1)}+\frac{e^2+1}{4}\ln|x|$. **2939.** $y=\frac{1}{2}x^2$. **2940.** $y=\frac{1}{2}x^2$. **2941.** $y=2e^x$. **2942.** $x=-\frac{3}{2}(y+2)^{\frac{2}{3}}$. **2943.** $y=e^x$. **2944.** $y^2=\frac{e}{e-1}+\frac{e^{-x}}{1-e}$. **2945.** $y=\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}-\frac{8}{3}$. **2946.** $y=\frac{3e^{3x}}{2+e^{3x}}$. **2947.** $y=\sec^2 x$. **2948.** $y=\sin x+1$ **2949.** $y=\frac{x^2}{4}-\frac{1}{2}$. **2950.** $x=-\frac{1}{2}e^{-y^2}$. **2951.** No solution. **2952.** $y=e^x$. **2953.** $y=2\ln|x|-\frac{2}{x}$. **2954.** $y=\frac{(x+C_1^2+1)^2}{2}+\frac{4}{3}C_1(x+1)^{\frac{3}{2}}+C_2$. Singular solution, $y=C$. **2955.** $y=C_1\frac{x^2}{2}+(C_1-C_1^2)x+C_2$. Singular solution, $y=\frac{(x+1)^3}{12}+C$. **2956.** $y=\frac{1}{12}(C_1+x)^4+C_2x+C_3$. **2957.** $y=C_1+C_2e^{C_1x}$; $y=1-e^x$; $y=-1+e^{-x}$; singular solution, $y=\frac{4}{C-x}$. **2958.** Circles. **2959.** $(x-C_1)^2-C_2y^2+kC_2^2=0$. **2960.** Catenary, $y=a\cosh\frac{x-x_0}{2}$. Circle, $(x-x_0)^2+y^2=a^2$. **2961.** Parabola, $(x-x_0)^2=2ay-a^2$. Cycloid, $x-x_0=a(t-\sin t)$, $y=a(1-\cos t)$. **2962.** $e^{ay+C_2}=\sec(ax+C_1)$. **2963.** Parabola. **2964.** $y=\frac{C_1}{2}\frac{H}{q}e^{\frac{q}{H}x}+\frac{1}{2C_1}\frac{H}{q}e^{-\frac{q}{H}x}+C_2=a\times$

$\times \cosh \frac{x+C}{a}+C_2$, where H is a constant horizontal tension, and $\frac{H}{q}=a$. **Hint.** The differential equation $\frac{d^2y}{dx^2}=\frac{q}{H}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$. **2965.** Equation of motion, $\frac{d^2s}{dt^2}=g(\sin\alpha-\mu\cos\alpha)$. Law of motion, $s=\frac{gt^2}{2}(\sin\alpha-\mu\cos\alpha)$ **2966.** $s=\frac{m}{k}\times$ $\times \ln\cosh\left(t\sqrt{g\frac{k}{m}}\right)$. **Hint.** Equation of motion, $m\frac{d^2s}{dt^2}=mg-k\left(\frac{ds}{dt}\right)^2$. **2967.** In 6.45 seconds. **Hint.** Equation of motion, $\frac{300}{g}\frac{d^2x}{dt^2}=-10\ v$. **2968.** a) No, b) yes, c) yes, d) yes, e) no, f) no, g) no, h) yes **2969.** a) $y''+y=0$; b) $y''-2y'+y=0$; c) $x^2y''-2xy'+2y=0$, d) $y'''-3y''+4y'-2y=0$ **2970.** $y=3x-5x^2+2x^3$. **2971.** $y=$ $=\frac{1}{x}(C_1\sin x+C_2\cos x)$. **Hint.** Use the substitution $y=y_1u$. **2972.** $y=C_1x+$ $+C_2\ln x$. **2973.** $y=A+Bx^2+x^3$. **2974.** $y=\frac{x^2}{3}+Ax+\frac{B}{x}$. **Hint.** Particular solutions of the homogeneous equation $y_1=x$, $y_2=\frac{1}{x}$. By the method of the variation of parameters we find: $C_1=\frac{x}{2}+A$, $C_2=-\frac{x^3}{6}+B$ **2975.** $y=A+$ $+B\sin x+C\cos x+\ln|\sec x+\tan x|+\sin x\ln|\cos x|-x\cos x$. **2976.** $y=C_1e^{2x}+$ $+C_2e^{3x}$ **2977.** $y=C_1e^{-3x}+C_2e^{3x}$. **2978.** $y=C_1+C_2e^{x}$ **2979.** $y=C_1\cos x+C_2\sin x$. **2980.** $y=e^x(C_1\cos x+C_2\sin x)$ **2981.** $y=e^{-2x}(C_1\cos 3x+C_2\sin 3x)$ **2982.** $y=$ $=(C_1+C_2x)e^{-x}$. **2983.** $y=e^{2x}(C_1e^{x\sqrt{2}}+C_2e^{-x\sqrt{2}})$. **2984.** If $k>0$, $y=$ $=C_1e^{x\sqrt{k}}+C_2e^{-x\sqrt{k}}$; if $k<0$, $y=C_1\cos\sqrt{-k}x+C_2\sin\sqrt{-k}x$. **2985.** $y=e^{-\frac{x}{2}}\left(C_1e^{\frac{\sqrt{5}}{2}x}+C_2e^{-\frac{\sqrt{5}}{2}x}\right)$ **2986.** $y=e^{\frac{x}{6}}\left(C_1\cos\frac{\sqrt{11}}{6}x+C_2\sin\frac{\sqrt{11}}{6}x\right)$. **2987.** $y=4e^x+e^{4x}$. **2988.** $y=e^{-x}$. **2989.** $y=\sin 2x$. **2990.** $y=1$. **2991.** $y=a\cosh\frac{x}{a}$. **2992.** $y=0$ **2993.** $y=C\sin\pi x$ **2994.** a) $xe^{2x}(Ax^2+Bx+C)$; b) $A\cos 2x+$ $+B\sin 2x$; c) $A\cos 2x+B\sin 2x+Cx^2e^{2x}$; d) $e^x(A\cos x+B\sin x)$, e) $e^x\times$ $\times(Ax^2+Bx+C)+xe^{2x}(Dx+E)$; f) $xe^x[(Ax^2+Bx+C)\cos 2x+(Dx^2+Ex+F)\times$ $\times\sin 2x]$ **2995.** $y=(C_1+C_2x)e^{2x}+\frac{1}{8}(2x^2+4x+3)$. **2996.** $y=e^{\frac{x}{2}}\left(C_1\cos\frac{x\sqrt{3}}{2}+\right.$ $\left.+C_2\sin\frac{x\sqrt{3}}{2}\right)+x^3+3x^2$. **2997.** $y=(C_1+C_2x)e^{-x}+\frac{1}{9}e^{2x}$.

2998. $y=C_1e^x+C_2e^{7x}+2$ **2999.** $y=C_1e^x+C_2e^{-x}+\frac{1}{2}xe^x$. **3000.** $y=C_1\cos x+$ $+C_2\sin x+\frac{1}{2}x\sin x$. **3001.** $y=C_1e^x+C_2e^{-2x}-\frac{2}{5}(3\sin 2x+\cos 2x)$. **3002.** $y=$ $=C_1e^{2x}+C_2e^{-3x}+x\left(\frac{x}{10}-\frac{1}{25}\right)e^{2x}$. **3003.** $y=(C_1+C_2x)e^x+\frac{1}{2}\cos x+\frac{x^2}{4}e^x-$ $-\frac{1}{8}e^{-x}$ **3004.** $y=C_1+C_2e^{-x}+\frac{1}{2}x+\frac{1}{20}(2\cos 2x-\sin 2x)$. **3005.** $y=e^x\times$ $\times(C_1\cos 2x+C_2\sin 2x)+\frac{x}{4}e^x\sin 2x$. **3006.** $y=\cos 2x+\frac{1}{3}(\sin x+\sin 2x)$.

3007. 1) $x=C_1\cos\omega t+C_2\sin\omega t+\frac{A}{\omega^2-p^2}\sin pt$; 2) $x=C_1\cos\omega t+C_2\sin\omega t-\frac{A}{2\omega}t\cdot\cos\omega t$. 3008. $y=C_1e^{3x}+C_2e^{4x}-xe^{4x}$. 3009. $y=C_1+C_2e^{2x}+\frac{x}{4}-\frac{x^2}{4}-\frac{x^3}{6}$. 3010. $y=e^x(C_1+C_2x+x^2)$. 3011. $y=C_1+C_2e^{2x}+\frac{1}{2}xe^{2x}-\frac{5}{2}x$. 3012. $y=C_1e^{-2x}+C_2e^{4x}-\frac{1}{9}e^x+\frac{1}{5}(3\cos 2x+\sin 2x)$. 3013. $y=C_1+C_2e^{-x}+e^x+\frac{5}{2}x^2-5x$. 3014. $y=C_1+C_2e^x-3xe^x-x-x^2$. 3015. $y=\left(C_1+C_2x+\frac{1}{2}x^2\right)\times e^{-x}+\frac{1}{4}e^x$. 3016. $y=(C_1\cos 3x+C_2\sin 3x)e^x+\frac{1}{37}(\sin 3x+6\cos 3x)+\frac{e^x}{9}$. 3017. $y=(C_1+C_2x+x^2)e^{2x}+\frac{x+1}{8}$. 3018. $y=C_1+C_2e^{3x}-\frac{1}{10}(\cos x+3\sin x)-\frac{x^2}{6}-\frac{x}{9}$. 3019. $y=\frac{1}{8}e^{2x}(4x+1)-\frac{x^3}{6}-\frac{x^2}{4}+\frac{x}{4}$. 3020. $y=C_1e^x+C_2e^{-x}-x\sin x-\cos x$. 3021. $y=C_1e^{-2x}+C_2e^{2x}-\frac{e^{2x}}{20}(\sin 2x+2\cos 2x)$. 3022. $y=C_1\cos 2x+C_2\sin 2x-\frac{x}{4}(3\sin 2x+2\cos 2x)+\frac{1}{4}$. 3023. $y=e^x(C_1\cos x+C_2\sin x-2x\cos x)$. 3024. $y=C_1e^x+C_2e^{-x}+\frac{1}{4}(x^2-x)e^x$. 3025. $y=C_1\cos 3x+C_2\sin 3x+\frac{1}{4}x\sin x-\frac{1}{16}\cos x+\frac{1}{54}(3x-1)e^{3x}$. 3026. $y=C_1e^{3x}+C_2e^{-x}+\frac{1}{9}\times(2-3x)+\frac{1}{16}(2x^2-x)e^{3x}$. 3027. $y=C_1+C_2e^{2x}-2xe^x-\frac{3}{4}x-\frac{3}{4}x^2$ 3028. $y=\left(C_1+C_2x+\frac{x^3}{6}\right)e^{2x}$. 3029. $y=C_1e^{-3x}+C_2e^x-\frac{1}{8}(2x^2+x)e^{-3x}+\frac{1}{16}\times(2x^2+3x)e^x$. 3030. $y=C_1\cos x+C_2\sin x+\frac{x}{4}\cos x+\frac{x^2}{4}\sin x-\frac{x}{8}\cos 3x+\frac{3}{32}\sin 3x$. **Hint.** Transform the product of cosines to the sum of cosines. 3031. $y=C_1e^{-x\sqrt{2}}+C_2e^{x\sqrt{2}}+xe^x\sin x+e^x\cos x$. 3032. $y=C_1\cos x+C_2\sin x+\cos x\ln\left|\cot\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$. 3033. $y=C_1\cos x+C_2\sin x+\sin x\cdot\ln\left|\tan\frac{x}{2}\right|$. 3034. $y=(C_1+C_2x)e^x+xe^x\ln|x|$. 3035. $y=(C_1+C_2x)e^{-x}+xe^{-x}\ln|x|$. 3036. $y=C_1\cos x+C_2\sin x+x\sin x+\cos x\ln|\cos x|$ 3037. $y=C_1\cos x+C_2\sin x-x\cos x+\sin x\ln|\sin x|$. 3038. a) $y=C_1e^x+C_2e^{-x}+(e^x+e^{-x})\times\operatorname{arc\,tan} e^x$; b) $y=C_1e^{x\sqrt{2}}+C_2e^{-x\sqrt{2}}+e^{x^2}$. 3040. Equation of motion, $\frac{2}{g}\left(\frac{d^2x}{dt^2}\right)=2-k(x+2)$; $(k=1)$; $T=2\pi\sqrt{\frac{2}{g}}$ sec. 3041. $x=\frac{2g\sin 30t-60\sqrt{g}\sin\sqrt{g}t}{g-900}$ cm. **Hint.** If x is reckoned from the position of rest of the load, then $\frac{4}{g}x''=4-k(x_0+x-y-l)$, where x_0 is the distance of the point of rest of the load from the initial point of suspension of the spring, l is the length of the spring at rest; therefore, $k(x_0-l)=4$, hence, $\frac{4}{g}\frac{d^2x}{dt^2}=-k(x-y)$, where $k=4$, $g=981$ cm/sec^2. 3042. $m\frac{d^2x}{dt^2}=k(b-x)-k(b+x)$

and $x = c\cos\left(t\sqrt{\frac{2k}{m}}\right)$. 3043. $6\frac{d^2s}{dt^2} = gs$; $t = \sqrt{\frac{6}{g}}\ln(6+\sqrt{35})$. 3044. a) $r = \frac{a}{2}(e^{\omega t}+e^{-\omega t})$; b) $r = \frac{v_0}{2\omega}(e^{\omega t}-e^{-\omega t})$ **Hint.** The differential equation of motion is $\frac{d^2r}{dt^2} = \omega^2 r$. 3045. $y = C_1 + C_2e^x + C_3e^{12x}$. 3046. $y = C_1 + C_2e^{-x} + C_3e^x$.

3047. $y = C_1e^{-x} + e^{\frac{x}{2}}\left(C_2\cos\frac{\sqrt{3}}{2}x + C_3\sin\frac{\sqrt{3}}{2}x\right)$

3048. $y = C_1 + C_2x + C_3e^{x\sqrt{2}} + C_4e^{-x\sqrt{2}}$ 3049. $y = e^x(C_1 + C_2x + C_3x^2)$.

3050. $y = e^x(C_1\cos x + C_2\sin x) + e^{-x}(C_3\cos x + C_4\sin x)$

3051. $y = (C_1 + C_2x)\cos 2x + (C_3 + C_4x)\sin 2x$

3052. $y = C_1 + C_2e^{-x} + e^{\frac{x}{2}}\left(C_3\cos\frac{\sqrt{3}}{2}x + C_4\sin\frac{\sqrt{3}}{2}x\right)$.

3053. $y = (C_1 + C_2x)e^{-x} + (C_3 + C_4x)e^x$.

3054. $y = C_1e^{ax} + C_2e^{-ax} + C_3\cos ax + C_4\sin ax$

3055. $y = (C_1 + C_2x)e^{\sqrt{3}x} + (C_3 + C_4x)e^{-\sqrt{3}x}$. 3056. $y = C_1 + C_2x + C_3\cos ax + C_4\sin ax$. 3057. $y = C_1 + C_2x + (C_3 + C_4x)e^{-x}$. 3058. $y = (C_1 + C_2x)\cos x + (C_3 + C_4x)\sin x$. 3059. $y = e^{-x}(C_1 + C_2x + \ldots + C_nx^{n-1})$.

3060. $y = C_1 + C_2x + \left(C_3 + C_4x + \frac{x^2}{2}\right)e^x$.

3061. $y = C_1 + C_2x + 12x^2 + 3x^3 + \frac{1}{2}x^4 + \frac{1}{20}x^5 + (C_3 + C_4x)e^x$.

3062. $y = C_1e^x + e^{-\frac{x}{2}}\left(C_2\cos\frac{\sqrt{3}}{2}x + C_3\sin\frac{\sqrt{3}}{2}x\right) - x^3 - 5$.

3063. $y = C_1 + C_2x + C_3x^2 + C_4e^{-x} + \frac{1}{1088}(4\cos 4x - \sin 4x)$

3064. $y = C_1e^{-x} + C_2 + C_3x + \frac{3}{2}x^2 - \frac{1}{3}x^3 + \frac{1}{12}x^4 + e^x\left(\frac{3}{2}x - \frac{15}{4}\right)$.

3065. $y = C_1e^{-x} + C_2\cos x + C_3\sin x + e^x\left(\frac{x}{4} - \frac{3}{8}\right)$.

3066. $y = C_1 + C_2\cos x + C_3\sin x + \sec x + \cos x\ln|\cos x| - \tan x\sin x + x\sin x$.

3067. $y = e^{-x} + e^{-\frac{x}{2}}\left(\cos\frac{\sqrt{3}}{2}x + \frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}x\right) + x - 2$.

3068. $y = (C_1 + C_2\ln x)\cdot\frac{1}{x}$. 3069. $y = C_1x^3 + \frac{C_2}{x}$.

3070. $y = C_1\cos(2\ln x) + C_2\sin(2\ln x)$.

3071. $y = C_1x + C_2x^2 + C_3x^3$. 3072. $y = C_1 + C_2(3x+2)^{-4/3}$.

3073. $y = C_1x^2 + \frac{C_2}{x}$. 3074. $y = C_1\cos(\ln x) + C_2\sin(\ln x)$.

3075. $y = C_1x^3 + C_2x^2 + \frac{1}{2}x$. 3076. $y = (x+1)^2[C_1 + C_2\ln(x+1)] + (x+1)^3$.

3077. $y = x(\ln x + \ln^2 x)$. 3078. $y = C_1\cos x + C_2\sin x$, $z = C_2\cos x - C_1\sin x$.

3079. $y = e^{-x}(C_1\cos x + C_2\sin x)$, $z = \frac{1}{5}e^{-x}[(C_2 - 2C_1)\cos x - (C_1 + 2C_2)\sin x]$.

3080. $y = (C_1 - C_2 - C_1x)e^{-2x}$, $z = (C_1x + C_2)e^{-2x}$.

3081. $x=C_1e^t+e^{-\frac{t}{2}}\left(C_2\cos\frac{\sqrt{3}}{2}t+C_3\sin\frac{\sqrt{3}}{2}t\right)$,

$y=C_1e^t+e^{-\frac{t}{2}}\left(\frac{C_3\sqrt{3}-C_2}{2}\cos\frac{\sqrt{3}}{2}t-\frac{C_2\sqrt{3}+C_3}{2}\sin\frac{\sqrt{3}}{2}t\right)$,

$z=C_1e^t+e^{-\frac{t}{2}}\left(\frac{-C_3\sqrt{3}-C_2}{2}\cos\frac{\sqrt{3}}{2}t+\frac{C_2\sqrt{3}-C_3}{2}\sin\frac{\sqrt{3}}{2}t\right)$.

3082. $x=C_1e^{-t}+C_2e^{2t}$, $y=C_3e^{-t}+C_2e^{2t}$, $z=-(C_1+C_3)e^{-t}+C_2e^{2t}$.

3083. $y=C_1+C_2e^{2x}-\frac{1}{4}(x^2+x)$, $z=C_2e^{2x}-C_1+\frac{1}{4}(x^2-x-1)$.

3084. $y=C_1+C_2x+2\sin x$, $z=-2C_1-C_2(2x+1)-3\sin x-2\cos x$.

3085. $y=(C_2-2C_1-2C_2x)e^{-x}-6x+14$, $z=(C_1+C_2x)e^{-x}+5x-9$;

$$C_1=9,\ C_2=4,$$

$y=14(1-e^{-x})-2x(3+4e^{-x})$, $z=-9(1-e^{-x})+x(5+4e^{-x})$.

3086. $x=10e^{2t}-8e^{3t}-e^t+6t-1$; $y=-20e^{2t}+8e^{3t}+3e^t+12t+10$.

3087. $y=\frac{2C_1}{(C_2-x)^2}$, $z=\frac{C_1}{C_2-x}$. **3088*.** a) $\frac{(x^2+y^2)\,y}{x}=C_1$, $\frac{z}{y}=C_2$; b) $\ln\sqrt{x^2+y^2}=\arctan\frac{y}{x}+C_1$, $\frac{z}{\sqrt{x^2+y^2}}=C_2$. **Hint.** Integrating the homogeneous equation $\frac{dx}{x-y}=\frac{dx}{x+y}$, we find the *first integral* $\ln\sqrt{x^2+p^2}=\operatorname{arc}\times\tan\frac{y}{x}+C_1$. Then, using the properties of derivative proportions, we have $\frac{dz}{z}=\frac{x\,dx}{x(x-y)}=\frac{y\,dy}{y(x+y)}=\frac{x\,dx+y\,dy}{x^2+y^2}$ Whence $\ln z=\frac{1}{2}\ln(x^2+y^2)+\ln C_2$ and, hence, $\frac{z}{\sqrt{x^2+y^2}}=C_2$; c) $x+y+z=0$, $x^2+y^2+z^2=6$. **Hint.** Applying the properties of derivative proportions, we have $\frac{dx}{y-z}=\frac{dy}{z-x}=\frac{dz}{x-y}=\frac{dx+dy+dz}{0}$; whence $dx+dy+dz=0$ and, consequently, $x+y+z=C_1$. Similarly, $\frac{x\,dx}{x(y-z)}=\frac{y\,dy}{y(z-x)}=\frac{z\,dz}{z(x-y)}=\frac{x\,dx+y\,dy+z\,dz}{0}$; $x\,dx+y\,dy+z\,dz=0$ and $x^2+y^2+z^2=C_2$. Thus, the integral curves are the circles $x+y+z=C_1$, $x^2+y^2+z^2=C_2$. From the initial conditions, $x=1$, $y=1$, $z=-2$, we will have $C_1=0$, $C_2=6$.

3089. $y=C_1x^2+\frac{C_2}{x}-\frac{x^2}{18}(3\ln^2 x-2\ln x)$,

$z=1-2C_1x+\frac{C_2}{x^2}+\frac{x}{9}(3\ln^2 x+\ln x-1)$.

3090. $y=C_1e^{x\sqrt{2}}+C_2e^{-x\sqrt{2}}+C_3\cos x+C_4\sin x+e^x-2x$,

$z=-C_1e^{x\sqrt{2}}-C_2e^{-x\sqrt{2}}-\frac{C_3}{4}\cos x-\frac{C_4}{4}\sin x-\frac{1}{2}e^x+x$.

3091. $x=\frac{v_0m\cos\alpha}{k}\left(1-e^{-\frac{k}{m}t}\right)$, $y=\frac{m}{k^2}(kv_0\sin\alpha+mg)\left(1-e^{-\frac{k}{m}t}\right)-\frac{mgt}{k}$.

Solution. $m\frac{dv_x}{dt}=-kv_x$; $m\frac{dv_y}{dt}=-kv_y-mg$ for the initial conditions: when

$t=0$, $x_0=y_0=0$, $v_{x_0}=v_0\cos\alpha$, $v_{y_0}=v_0\sin\alpha$. Integrating, we obtain $v_x=$ $=v_0\cos\alpha e^{-\frac{k}{m}t}$, $kv_y+mg=(kv_0\sin\alpha+mg)e^{-\frac{k}{m}t}$. **3092.** $x=a\cos\frac{k}{\sqrt{m}}t$, $y=$ $=\frac{v_0\sqrt{m}}{k}\sin\frac{k}{\sqrt{m}}t$, $\frac{x^2}{a^2}+\frac{k^2y^2}{mv_0^2}=1$. **Hint.** The differential equations of motion: $m\frac{d^2x}{dt^2}=-k^2x$, $m\frac{d^2y}{dt^2}=-k^2y$.

3093. $y=-2-2x-x^2$. **3094.** $y=\left(y_0+\frac{1}{4}\right)e^{2(x-1)}-\frac{1}{2}x+\frac{1}{4}$.

3095. $y=\frac{1}{2}+\frac{1}{4}x+\frac{1}{8}x^2+\frac{1}{16}x^3+\frac{9}{32}x^4+\frac{21}{320}x^5+\ldots$

3096. $y=\frac{1}{3}x^3-\frac{1}{7\cdot 9}x^7+\frac{2}{7\cdot 11\cdot 27}x^{11}-\ldots$

3097. $y=x+\frac{x^2}{1\cdot 2}+\frac{x^3}{2\cdot 3}+\frac{x^4}{3\cdot 4}+\ldots$; the series converges for $-1\leqslant x\leqslant 1$.

3098. $y=x-\frac{x^2}{(1!)^2\cdot 2}+\frac{x^3}{(2!)^2\cdot 3}-\frac{x^4}{(3!)^2\cdot 4}+\ldots$; the series converges for $-\infty<$ $<x<+\infty$. **Hint.** Use the method of undetermined coefficients.

3099. $y=1-\frac{1}{3!}x^3+\frac{1\cdot 4}{6!}x^6-\frac{1\cdot 4\cdot 7}{9!}x^9\ldots$; the series converges for $-\infty<x<+\infty$.

3100. $y=\frac{\sin x}{x}$. **Hint.** Use the method of undetermined coefficients.

3101. $y=1-\frac{x^2}{2^2}+\frac{x^4}{2^2\cdot 4^2}-\frac{x^6}{2^2\cdot 4^2\cdot 6^2}+\ldots$; the series converges for $|x|<\infty$. **Hint.** Use the method of undetermined coefficients. **3102.** $x=a\left(1-\frac{1}{2!}t^2+\right.$ $\left.+\frac{2}{4!}t^4-\frac{9}{6!}t^6+\frac{55}{8!}t^8-\ldots\right)$. **3103.** $u=A\cos\frac{a\pi t}{l}\sin\frac{\pi x}{l}$. **Hint.** Use the conditions: $u(0,t)=0$, $u(l,t)=0$, $u(x,0)=A\sin\frac{\pi x}{l}$, $\frac{\partial u(x,0)}{\partial t}=0$.

3104. $u=\frac{2l}{\pi^2 a}\sum_{n=1}^{\infty}\frac{1}{n^2}(1-\cos n\pi)\sin\frac{n\pi at}{l}\sin\frac{n\pi x}{l}$. **Hint.** Use the conditions: $u(0,t)=0$, $u(l,t)=0$, $u(x,0)=0$, $\frac{\partial u(x,0)}{\partial t}=1$.

3105. $u=\frac{8h}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{2}\cos\frac{n\pi at}{l}\sin\frac{n\pi x}{l}$. **Hint.** Use the conditions:

$$\frac{\partial u(x,0)}{\partial t}=0,\quad u(0,t)=0,\quad u(l,t)=0,\quad u(x,0)=\begin{cases}\frac{2hx}{l} & \text{for } 0<x\leqslant\frac{l}{2},\\ 2h\left(1-\frac{x}{l}\right) & \text{for } \frac{l}{2}<x<l.\end{cases}$$

3106. $u=\sum_{n=0}^{\infty}A_n\cos\frac{(2n+1)a\pi t}{2l}\sin\frac{(2n+1)\pi x}{2t}$, where the coefficients $A_n=$

$= \frac{2}{l}\int_0^l \frac{x}{l}\sin\frac{(2n+1)\pi x}{2l}\,dx$. **Hint.** Use the conditions

$$u(0,\ t)=0,\quad \frac{\partial u(l,\ t)}{\partial x}=0,\ u(x,\ 0)\frac{x}{l},\quad \frac{\partial u(x,\ 0)}{\partial t}=0.$$

3107. $u=\frac{400}{\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^3}(1-\cos n\pi)\sin\frac{n\pi x}{100}\cdot e^{-\frac{a^2n^2\pi^2 t}{100^2}}$.

Hint. Use the conditions: $u(0,\ t)=0$, $u(100,\ t)=0$, $u(x,\ 0)=0.01\,x(100-x)$.

Chapter X

3108. a) $\leqslant 1''$; $\leqslant 0.0023\%$; b) $\leqslant 1$ mm; $\leqslant 0.26\%$; c) $\leqslant 1$ gm; $\leqslant 0.0016\%$. **3109.** a) $\leqslant 0.05$; $\leqslant 0.021\%$; b) $\leqslant 0.0005$; $\leqslant 1.45\%$; c) $\leqslant 0.005$; $\leqslant 0.16\%$. **3110.** a) two decimals; $48\cdot 10^3$ or $49\cdot 10^3$, since the number lies between 47,877 and 48,845; b) two decimals; 15; c) one decimal; $6\cdot 10^2$. For practical purposes there is sense in writing the result in the form $(5.9\pm 0.1)\cdot 10^2$. **3111.** a) 29.5; b) $1.6\cdot 10^2$; c) 43.2. **3112.** a) 84.2; b) 18.5 or 18.47 ± 0.01; c) the result of subtraction does not have any correct decimals, since the difference is equal to one hundredth with a possible absolute error of one hundredth. **3113*.** 1.8 ± 0.3 cm^2. **Hint.** Use the formula for increase in area of a square. **3114.** a) 30.0 ± 0.2; b) 43.7 ± 0.1; c) 0.3 ± 0.1. **3115.** 19.9 ± 0.1 m^2. **3116.** a) 1.1295 ± 0.0002; b) 0.120 ± 0.006; c) the quotient may vary between 48 and 62. Hence, not a single decimal place in the quotient may be considered certain. **3117.** 0.480. The last digit may vary by unity. **3118.** a) 0.1729; b) $277\cdot 10^3$; c) 2. **3119.** $(2.05\pm 0.01)\cdot 10^3$ cm^2. **3120.** a) 1.648; b) 4.025 ± 0.001; c) 9.006 ± 0.003. **3121.** $4.01\cdot 10^3$ cm^2. Absolute error, 6.5 cm^2. Relative error, 0.16%. **3122.** The side is equal to $13.8+0.2$ cm; $\sin\alpha=0.44\pm 0.01$, $\alpha=26°15'\pm$ $\pm 35'$. **3123.** 2.7 ± 0.1. **3124.** 0.27 ampere **3125.** The length of the pendulum should be measured to within 0.3 cm; take the numbers π and q to three decimals (on the principle of equal effects). **3126.** Measure the radii and the generatrix with relative error 1/300. Take the number π to three decimal places (on the principle of equal effects). **3127.** Measure the quantity l to within 0.2%, and s to within 0.7% (on the principle of equal effects).

3128.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1	3	7	−2	−6	14	−23
2	10	5	−8	8	−9	
3	15	−3	0	−1		
4	12	−3	−1			
5	9	−4				
6	5					

3129.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1	—4	—12	32	48
3	—16	20	80	48
5	4	100	128	48
7	104	228	176	
9	332	404		
11	736			

3130.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	0	—4	—42	—24	24
1	—4	—46	—66	0	24
2	—50	—112	—66	24	24
3	—162	—178	—42	48	24
4	—340	—220	6	72	24
5	—560	—214	78	96	24
6	—774	—136	174	120	24
7	—910	38	294	144	
8	—872	332	438		
9	—540	770			
10	230				

Hint. Compute the first five values of y and, after obtaining $\Delta^4 y_0 = 24$, repeat the number 24 throughout the column of fourth differences. After this the remaining part of the table is filled in by the operation of addition (moving from right to left).

3131. a) 0.211; 0.389; 0.490; 0.660; b) 0.229; 0.399; 0.491; 0.664. **3132.** 0 1822; 0.1993; 0.2165; 0.2334; 0.2503. **3133.** $1+x+x^2+x^3$. **3134.** $y=\frac{1}{96}x^4-\frac{11}{48}x^3+\frac{65}{24}x^2-\frac{85}{12}x+8$; $y\approx 22$ for $x=5.5$; $y=20$ for $x\approx 5.2$. **Hint.** When computing x for $y=20$ take $y_0=11$. **3135.** The interpolating polynomial is $y=x^2-10x+1$; $y=1$ when $x=0$. **3136.** 158 kgf (approximately). **3137.** a) $y(0.5)=-1$, $y(2)=11$; b) $y(0.5)=-\frac{15}{16}$, $y(2)=-3$. **3138.** -1.325 **3139.** 1.01. **3140.** —1.86; —0.25; 2.11. **3141.** 2.09. **3142.** 2 45 and 0 019. **3143.** 0.31 and 4 **3144.** 2.506. **3145.** 0.02. **3146.** 0 24. **3147.** 1 27 **3148.** —1.88; 0 35; 1 53 **3149.** 1.84. **3150.** 1.31 and —0.67. **3151.** 7.13. **3152.** 0.165. **3153.** 1.73 and 0. **3154.** 1.72. **3155.** 1 38 **3156.** $x=0.83$; $y=0$ 56; $x=-0.83$; $y=-0.56$ **3157.** $x=1.67$; $y=1$ 22. **3158.** 4 493. **3159.** ± 1 1997 **3160.** By the trapezoidal formula, 11.625; by Simpson's formula, 11 417. **3161.** —0 995; —1; 0.005; 0.5%; $\Delta=0.005$. **3162.** 0.3068; $\Delta=1.3\cdot 10^{-5}$. **3163.** 0 69 **3164.** 0.79. **3165.** 0.84. **3166.** 0.28. **3167.** 0.10. **3168.** 1 61. **3169.** 1.85 **3170.** 0.09. **3171.** 0.67. **3172.** 0.75. **3173.** 0.79. **3174.** 4.93. **3175.** 1 29. **Hint.** Make use of the parametric equation of the ellipse $x=\cos t$, $y=0.6222\sin t$ and transform the formula of the arc length to the form $\int_0^{\frac{\pi}{2}}\sqrt{1-\varepsilon^2\cos^2 t}\cdot dt$, where ε is the eccentricity of the ellipse. **3176.** $y_1(x)=\frac{x^3}{3}$, $y_2(x)=\frac{x^3}{3}+\frac{x^7}{63}$, $y_3(x)=\frac{x^3}{3}+\frac{x^7}{63}+\frac{2x^{11}}{2079}+\frac{x^{15}}{59535}$. **3177.** $y_1(x)=\frac{x^2}{2}-x+1$, $y_2(x)=\frac{x^3}{6}+\frac{3x^2}{2}-x+1$, $y_3(x)=\frac{x^4}{12}-\frac{x^3}{6}+\frac{3x^2}{2}-x+1$; $z_1(x)=3x-2$, $z_2(x)=\frac{x^3}{6}-2x^2+3x-2$, $z_3(x)=\frac{7x^3}{6}-2x^2+3x-2$. **3178.** $y_1(x)=x$, $y_2(x)=x-\frac{x^3}{6}$, $y_3(x)=x-\frac{x^3}{6}+\frac{x^5}{120}$.
3179. $y(1)=3.36$. **3180.** $y(2)=0.80$. **3181.** $y(1)=3.72$; $z(1)=2.72$ **3182.** $y=1.80$. **3183.** 3.15. **3184.** 0.14. **3185.** $y(0.5)=3$ 15; $z(0$ 5$)=-3$ 15. **3186.** $y(0.5)=0.55$; $z(0$ 5$)=-0.18$. **3187.** 1.16. **3188.** 0 87. **3189.** $x(\pi)=3.58$; $x'(\pi)=0.79$. **3190.** $429+1739\cos x-1037\sin x-6321\cos 2x+1263\sin 2x-1242\cos 3x-33\sin 3x$. **3191.** 6 $49-1$ $96\cos x+2.14\sin x-1.68\cos 2x+0.53\sin 2x-1.13\cos 3x+0.04\sin 3x$. **3192.** $0.960+0.851\cos x+0.915\sin x+0.542\cos 2x+0.620\sin 2x+0.271\cos 3x+0.100\sin 3x$. **3193.** a) 0 $608\sin x+0.076\sin 2x+0.022\sin 3x$; b) $0.338+0.414\cos x+0.111\cos 2x+0.056\cos 3x$.

APPENDIX

I. Greek Alphabet

Alpha—Αα
Beta—Ββ
Gamma—Γγ
Delta—Δδ
Epsilon—Εε
Zeta—Ζζ
Eta—Ηη
Theta—Θθ
Iota—Ιι
Kappa—Κκ
Lambda—Λλ
Mu—Μμ
Nu—Νν
Xi—Ξξ
Omicron—Οο
Pi—Ππ
Rho—Ρρ
Sigma—Σσ
Tau—Ττ
Upsilon—Υυ
Phi—Φφ
Chi—Χχ
Psi—Ψψ
Omega—Ωω

II. Some Constants

Quantity	x	$\log x$	Quantity	x	$\log x$
π	3.14159	0.49715	$\frac{1}{e}$	0.36788	$\bar{1}.56571$
2π	6.28318	0.79818	e^2	7.38906	0.86859
$\frac{\pi}{2}$	1.57080	0.19612	$\sqrt{e}$	1.64872	0.21715
$\frac{\pi}{4}$	0.78540	$\bar{1}.89509$	$\sqrt[3]{e}$	1.39561	0.14476
$\frac{1}{\pi}$	0.31831	$\bar{1}.50285$	$M=\log e$	0.43429	$\bar{1}.63778$
π^2	9.86960	0.99430	$\frac{1}{M}=\ln 10$	2.30258	0.36222
$\sqrt{\pi}$	1.77245	0.24857	1 radian	57°17′45″	
$\sqrt[3]{\pi}$	1.46459	0.16572	arc 1°	0.01745	$\bar{2}.24188$
e	2.71828	0.43429	g	9.81	0.99167

III. Inverse Quantities, Powers, Roots, Logarithms

x	$\frac{1}{x}$	x^2	x^3	$\sqrt{x}$	$\sqrt{10x}$	$\sqrt[3]{x}$	$\sqrt[3]{10x}$	$\sqrt[3]{100x}$	log x (mantissas)	ln x
1.0	1.000	1.000	1.000	1.000	3.162	1.000	2.154	4.642	0000	0.0000
1.1	0.909	1.210	1.331	1.049	3.317	1.032	2 224	4.791	0414	0.0953
1.2	0.833	1.440	1.728	1.095	3.464	1.063	2.289	4.932	0792	0.1823
1.3	0.769	1.690	2.197	1.140	3.606	1.091	2.351	5.066	1139	0.2624
1.4	0.714	1.960	2.744	1.183	3.742	1.119	2.410	5.192	1461	0.3365
1.5	0.667	2.250	3.375	1.225	3.873	1.145	2.466	5.313	1761	0.4055
1.6	0.625	2.560	4.096	1.265	4.000	1.170	2.520	5.429	2041	0.4700
1.7	0.588	2.890	4.913	1.304	4.123	1 193	2.571	5.540	2304	0.5306
1.8	0.556	3.240	5.832	1.342	4.243	1.216	2.621	5.646	2553	0 5878
1.9	0.526	3.610	6.859	1 378	4.359	1.239	2.668	5.749	2788	0.6419
2.0	0.500	4.000	8.000	1.414	4.472	1.260	2.714	5.848	3010	0.6931
2.1	0.476	4.410	9.261	1.449	4.583	1.281	2.759	5.944	3222	0.7419
2.2	0.454	4.840	10.65	1.483	4.690	1.301	2 802	6.037	3424	0.7885
2.3	0.435	5.290	12.17	1.517	4.796	1.320	2.844	6.127	3617	0.8329
2.4	0.417	5.760	13.82	1.549	4.899	1.339	2.884	6.214	3802	0.8755
2.5	0.400	6.250	15.62	1.581	5.000	1.357	2.924	6.300	3979	0.9163
2.6	0.385	6.760	17.58	1.612	5.099	1.375	2.962	6.383	4150	0.9555
2.7	0.370	7.290	19.68	1.643	5.196	1.392	3.000	6.463	4314	0.9933
2.8	0.357	7.840	21.95	1.673	5.292	1.409	3.037	6.542	4472	1.0296
2.9	0.345	8.410	24.39	1.703	5.385	1.426	3.072	6.619	4624	1 0647
3.0	0.333	9.000	27.00	1.732	5.477	1.442	3.107	6.694	4771	1 0986
3.1	0.323	9.610	29.79	1.761	5.568	1.458	3.141	6.768	4914	1.1314
3.2	0.312	10.24	32.77	1.789	5.657	1.474	3.175	6.840	5051	1.1632
3.3	0.303	10.89	35.94	1.817	5.745	1.489	3.208	6.910	5185	1.1939
3.4	0.294	11.56	39.30	1.844	5.831	1.504	3.240	6.980	5315	1.2238
3.5	0.286	12.25	42.88	1.871	5.916	1.518	3.271	7.047	5441	1.2528
3.6	0.278	12.96	46.66	1.897	6.000	1.533	3.302	7.114	5563	1.2809
3.7	0.270	13.69	50.65	1.924	6.083	1.547	3.332	7.179	5682	1.3083
3.8	0.263	14.44	54.87	1.949	6.164	1.560	3.362	7.243	5798	1.3350
3.9	0.256	15.21	59.32	1.975	6.245	1.574	3.391	7.306	5911	1.3610
4.0	0.250	16.00	64.00	2.000	6.325	1.587	3.420	7.368	6021	1.3863
4.1	0.244	16.81	68.92	2.025	6.403	1.601	3.448	7.429	6128	1.4110
4.2	0.238	17.64	74.09	2.049	6.481	1.613	3.476	7.489	6232	1 4351
4.3	0.233	18.49	79.51	2.074	6.557	1.626	3.503	7.548	6335	1.4586
4.4	0.227	19.36	85.18	2.098	6.633	1.639	3.530	7.606	6435	1.4816
4.5	0.222	20.25	91.12	2.121	6.708	1.651	3.557	7.663	6532	1.5041
4.6	0.217	21.16	97.34	2.145	6.782	1.663	3.583	7.719	6628	1.5261
4.7	0.213	22.09	103.8	2.168	6.856	1.675	3.609	7.775	6721	1.5476
4.8	0.208	23.04	110.6	2.191	6.928	1.687	3.634	7.830	6812	1.5686
4.9	0.204	24.01	117.6	2.214	7.000	1.698	3.659	7.884	6902	1.5892
5.0	0.200	25.00	125.0	2.236	7.071	1.710	3.684	7.937	6990	1.6094
5.1	0.196	26.01	132.7	2.258	7.141	1.721	3.708	7.990	7076	1.6292
5.2	0.192	27.04	140.6	2.280	7.211	1.732	3.733	8.041	7160	1.6487
5.3	0.189	28.09	148.9	2 302	7.280	1.744	3.756	8.093	7243	1.6677
5.4	0.185	29.16	157.5	2.324	7.348	1.754	3.780	8.143	7324	1.6864

Continued

x	$\frac{1}{x}$	x^2	x^3	$\sqrt{x}$	$\sqrt{10x}$	$\sqrt[3]{x}$	$\sqrt[3]{10x}$	$\sqrt[3]{100x}$	log x (mantissas)	ln x
5.5	0 182	30 25	166 4	2.345	7.416	1.765	3.803	8.193	7404	1.7047
5 6	0.179	31 36	175.6	2 366	7 483	1 776	3.826	8.243	7482	1 7228
5.7	0.175	32 49	185 2	2 387	7 550	1.786	3.849	8.291	7559	1.7405
5.8	0.172	33.64	195.1	2 408	7 616	1.797	3.871	8.340	7634	1.7579
5.9	0 169	34 81	205.4	2 429	7.681	1.807	3 893	8 387	7709	1.7750
6.0	0.167	36.00	216.0	2.449	7 746	1.817	3.915	8.434	7782	1.7918
6.1	0.164	37 21	227.0	2.470	7.810	1 827	3.936	8 481	7853	1.8083
6.2	0 161	38.44	238 3	2 490	7 874	1.837	3.958	8 527	7924	1.8245
6.3	0 159	39.69	250 0	2.510	7.937	1.847	3.979	8 573	7993	1.8405
6 4	0.156	40 96	262.1	2.530	8.000	1.857	4.000	8 618	8062	1.8563
6.5	0.154	42.25	274 6	2.550	8.062	1.866	4 021	8.662	8129	1.8718
6 6	0.151	43.56	287 5	2 569	8.124	1 876	4.041	8.707	8195	1.8871
6 7	0.149	44 89	300 8	2 588	8.185	1 885	4 062	8 750	8261	1 9021
6 8	0.147	46 24	314 4	2 608	8.246	1.895	4.082	8 794	8325	1 9169
6 9	0 145	47 61	328 5	2.627	8 307	1.904	4.102	8.837	8388	1 9315
7.0	0.143	49.00	343 0	2.646	8.367	1.913	4.121	8 879	8451	1 9459
7.1	0.141	50.41	357.9	2 665	8 426	1.922	4.141	8 921	8513	1.9601
7.2	0.139	51.84	373.2	2.683	8.485	1.931	4.160	8.963	8573	1.9741
7.3	0 137	53.29	389.0	2.702	8.544	1.940	4.179	9.004	8633	1.9879
7.4	0.135	54 76	405.2	2.720	8 602	1.949	4.198	9.045	8692	2.0015
7.5	0.133	56.25	421.9	2.739	8.660	1.957	4.217	9.086	8751	2.0149
7.6	0.132	57.76	439 0	2 757	8.718	1.966	4.236	9.126	8808	2.0281
7.7	0 130	59 29	456 5	2 775	8.775	1.975	4.254	9.166	8865	2.0412
7.8	0 128	60.84	474 6	2.793	8.832	1.983	4 273	9.205	8921	2.0541
7.9	0 127	62.41	493.0	2.811	8.888	1 992	4 291	9.244	8976	2.0669
8.0	0 125	64.00	512 0	2.828	8.944	2.000	4.309	9.283	9031	2.0794
8.1	0 123	65.61	531.4	2 846	9.000	2 008	4.327	9.322	9085	2.0919
8.2	0.122	67.24	551.4	2.864	9 055	2 017	4.344	9 360	9138	2.1041
8.3	0.120	68.89	571.8	2.881	9.110	2 025	4.362	9.398	9191	2.1163
8 4	0 119	70.56	592.7	2.898	9.165	2.033	4 380	9.435	9243	2.1282
8.5	0.118	72.25	614.1	2.915	9.220	2.041	4.397	9.473	9294	2.1401
8.6	0.116	73.96	636.1	2 933	9.274	2 049	4.414	9 510	9345	2.1518
8.7	0.115	75.69	658.5	2.950	9.327	2.057	4.431	9.546	9395	2.1633
8.8	0.114	77.44	681.5	2.966	9.381	2.065	4.448	9.583	9445	2.1748
8 9	0 112	79 21	705.0	2 983	9.434	2 072	4.465	9 619	9494	2.1861
9.0	0.111	81.00	729.0	3.000	9.487	2.080	4.481	9.655	9542	2 1972
9.1	0.110	82.81	753.6	3.017	9.539	2.088	4.498	9.691	9590	2.2083
9.2	0.109	84.64	778.7	3.033	9.592	2.095	4.514	9.726	9638	2.2192
9.3	0.108	86.49	804.4	3.050	9.644	2.103	4.531	9.761	9685	2.2300
9.4	0.106	88.36	830 6	3 066	9.695	2.110	4.547	9.796	9731	2 2407
9.5	0.105	90.25	857.4	3.082	9.747	2.118	4.563	9.830	9777	2.2513
9.6	0.104	92.16	884.7	3.098	9.798	2.125	4.579	9 865	9823	2.2618
9.7	0.103	94.09	912 7	3.114	9.849	2.133	4.595	9 899	9868	2 2721
9.8	0.102	96.04	941.2	3.130	9.899	2.140	4.610	9.933	9912	2.2824
9.9	0.101	98.01	970.3	3.146	9.950	2.147	4.626	9.967	9956	2.2925
10.0	0.100	100.00	1000.0	3.162	10.000	2.154	4.642	10.000	0000	2.3026

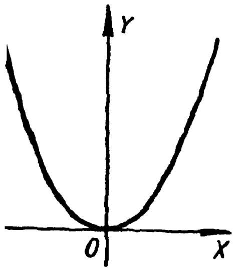
IV. Trigonometric Functions

$x°$	x (radians)	sin x	tan x	cot x	cos x		
0	0.0000	0.0000	0.0000	∞	1.0000	1.5708	90
1	0.0175	0.0175	0.0175	57 29	0.9998	1.5533	89
2	0.0349	0.0349	0.0349	28.64	0.9994	1 5359	88
3	0.0524	0.0523	0.0524	19.08	0.9986	1.5184	87
4	0.0698	0.0698	0.0699	14 30	0.9976	1.5010	86
5	0.0873	0.0872	0.0875	11.43	0.9962	1.4835	85
6	0.1047	0.1045	0.1051	9.514	0.9945	1.4661	84
7	0.1222	0.1219	0.1228	8.144	0.9925	1.4486	83
8	0.1396	0.1392	0.1405	7.115	0.9903	1 4312	82
9	0.1571	0.1564	0 1584	6 314	0.9877	1 4137	81
10	0.1745	0.1736	0.1763	5.671	0.9848	1 3963	80
11	0.1920	0.1908	0.1944	5.145	0.9816	1.3788	79
12	0.2094	0.2079	0.2126	4.705	0 9781	1 3614	78
13	0.2269	0.2250	0.2309	4.331	0.9744	1.3439	77
14	0.2443	0.2419	0.2493	4.011	0.9703	1.3265	76
15	0.2618	0.2588	0.2679	3.732	0.9659	1.3090	75
16	0.2793	0.2756	0.2867	3.487	0.9613	1.2915	74
17	0.2967	0.2924	0.3057	3.271	0.9563	1.2741	73
18	0.3142	0.3090	0.3249	3.078	0.9511	1 2566	72
19	0.3316	0.3256	0.3443	2.904	0.9455	1 2392	71
20	0.3491	0.3420	0.3640	2.747	0 9397	1 2217	70
21	0.3665	0.3584	0.3839	2.605	0.9336	1.2043	69
22	0.3840	0.3746	0.4040	2.475	0.9272	1.1868	68
23	0.4014	0.3907	0.4245	2.356	0.9205	1.1694	67
24	0.4189	0.4067	0.4452	2.246	0.9135	1.1519	66
25	0.4363	0.4226	0.4663	2.145	0.9063	1 1345	65
26	0.4538	0 4384	0 4877	2.050	0.8988	1.1170	64
27	0.4712	0.4540	0.5095	1.963	0.8910	1.0996	63
28	0.4887	0.4695	0.5317	1.881	0.8829	1.0821	62
29	0.5061	0 4848	0.5543	1.804	0.8746	1.0647	61
30	0.5236	0.5000	0.5774	1 732	0.8660	1.0472	60
31	0.5411	0.5150	0.6009	1.6643	0.8572	1.0297	59
32	0.5585	0.5299	0.6249	1.6003	0.8480	1.0123	58
33	0.5760	0.5446	0 6494	1.5399	0.8387	0.9948	57
34	0.5934	0.5592	0.6745	1.4826	0 8290	0.9774	56
35	0.6109	0.5736	0.7002	1.4281	0.8192	0.9599	55
36	0.6283	0.5878	0.7265	1.3764	0.8090	0.9425	54
37	0.6458	0.6018	0.7536	1.3270	0.7986	0.9250	53
38	0.6632	0.6157	0.7813	1.2799	0.7880	0.9076	52
39	0.6807	0.6293	0.8098	1.2349	0.7771	0.8901	51
40	0.6981	0.6428	0.8391	1.1918	0.7660	0.8727	50
41	0.7156	0.6561	0.8693	1.1504	0.7547	0.8552	49
42	0.7330	0.6691	0.9004	1.1106	0.7431	0.8378	48
43	0.7505	0.6820	0 9325	1.0724	0.7314	0.8203	47
44	0.7679	0.6947	0.9657	1.0355	0.7193	0.8029	46
45	0.7854	0.7071	1.0000	1.0000	0.7071	0.7854	45
		cos x	cot x	tan x	sin x	x (radians)	$x°$

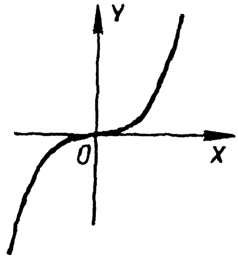
V. Exponential, Hyperbolic and Trigonometric Functions

x	e^x	e^{-x}	sinh x	cosh x	tanh x	sin x	cos x
0.0	1.0000	1.0000	0.0000	1.0000	0.0000	0.0000	1.0000
0.1	1.1052	0.9048	0.1002	1.0050	0.0997	0.0998	0.9950
0.2	1.2214	0.8187	0.2013	1.0201	0.1974	0.1987	0.9801
0.3	1.3499	0.7408	0.3045	1.0453	0.2913	0.2955	0.9553
0.4	1.4918	0.6703	0.4108	1.0811	0.3799	0.3894	0.9211
0.5	1.6487	0.6065	0.5211	1.1276	0.4621	0.4794	0.8776
0.6	1.8221	0.5488	0.6367	1.1855	0.5370	0.5646	0.8253
0.7	2.0138	0.4966	0.7586	1.2552	0.6044	0.6442	0.7648
0.8	2.2255	0.4493	0.8881	1.3374	0.6640	0.7174	0.6967
0.9	2.4596	0.4066	1.0265	1.4331	0.7163	0.7833	0.6216
1.0	2.7183	0.3679	1.1752	1.5431	0.7616	0.8415	0.5403
1.1	3.0042	0.3329	1.3356	1.6685	0.8005	0.8912	0.4536
1.2	3.3201	0.3012	1.5095	1.8107	0.8337	0.9320	0.3624
1.3	3.66 3	0.2725	1.6981	1.9709	0.8617	0.9636	0.2675
1.4	4.0552	0.2466	1.9043	2.1509	0.8854	0.9854	0.1700
1.5	4.4817	0.2231	2.1293	2.3524	0.9051	0.9975	0.0707
1.6	4.9530	0.2019	2.3756	2.5775	0.9217	0.9996	−0.0292
1.7	5.4739	0.1827	2.6456	2.8283	0.9354	0.9917	−0.1288
1.8	6.0496	0.1653	2.9422	3.1075	0.9468	0.9738	−0.2272
1.9	6.6859	0.1496	3.2682	3.4177	0.9562	0.9463	−0.3233
2.0	7.3891	0.1353	3.6269	3.7622	0.9640	0.9093	−0.4161
2.1	8.1662	0.1225	4.0219	4.1443	0.9704	0.8632	−0.5048
2.2	9.0250	0.1108	4.4571	4.5679	0.9757	0.8085	−0.5885
2.3	9.9742	0.1003	4.9370	5.0372	0.9801	0.7457	−0.6663
2.4	11.0232	0.0907	5.4662	5.5569	0.9837	0.6755	−0.7374
2.5	12.1825	0.0821	6.0502	6.1323	0.9866	0.5985	−0.8011
2.6	13.4637	0.0743	6.6947	6.7690	0.9890	0.5155	−0.8569
2.7	14.8797	0.0672	7.4063	7.4735	0.9910	0.4274	−0.9041
2.8	16.4446	0.0608	8.1919	8.2527	0.9926	0.3350	−0.9422
2.9	18.1741	0.0550	9.0596	9.1146	0.9940	0.2392	−0.9710
3.0	20.0855	0.0498	10.0179	10.0677	0.9950	0.1411	−0.9900
3.1	22.1979	0.0450	11.0764	11.1215	0.9959	0.0416	−0.9991
3.2	24.5325	0.0408	12.2459	12.2366	0.9967	−0.0584	−0.9983
3.3	27.1126	0.0369	13.5379	13.5748	0.9973	−0.1577	−0.9875
3.4	29.9641	0.0334	14.9654	14.9987	0.9978	−0.2555	−0.9668
3.5	33.1154	0.0302	16.5426	16.5728	0.9982	−0.3508	−0.9365

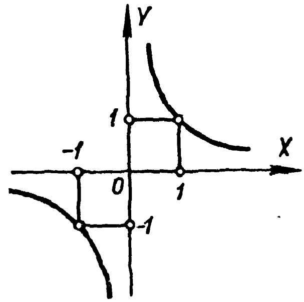
VI. Some Curves (for Reference)



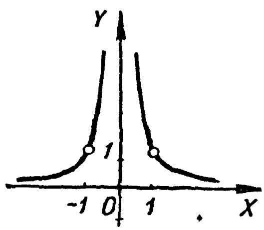
1. Parabola,
$y=x^2$.



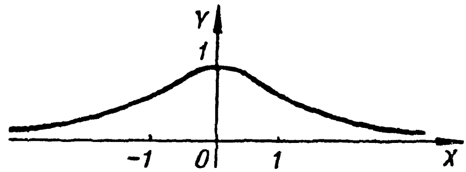
2. Cubic parabola,
$y=x^3$.



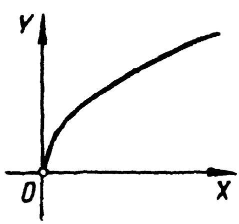
3. Rectangular hyperbola,
$y=\frac{1}{x}$.



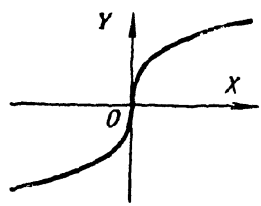
4. Graph of a fractional function,
$y=\frac{1}{x^2}$.



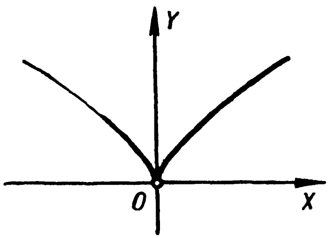
5. The witch of Agnesi,
$y=\frac{1}{1+x^2}$.



6. Parabola (upper branch),
$y=\sqrt{x}$.



7. Cubic parabola,
$y=\sqrt[3]{x}$.

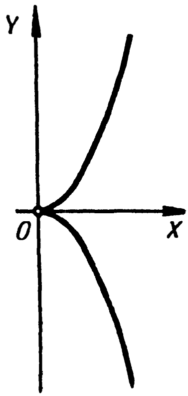


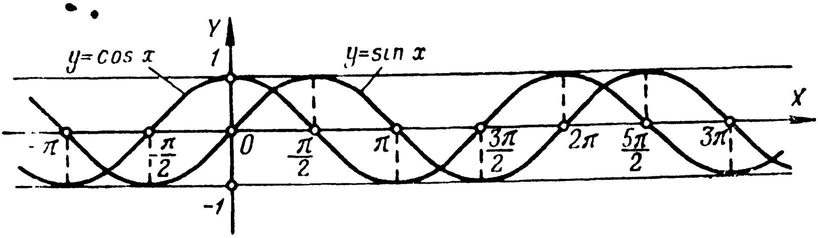
8*a*. Neile's parabola,

$y = x^{\frac{2}{3}}$ or $\begin{cases} x = t^3, \\ x = -t^2. \end{cases}$

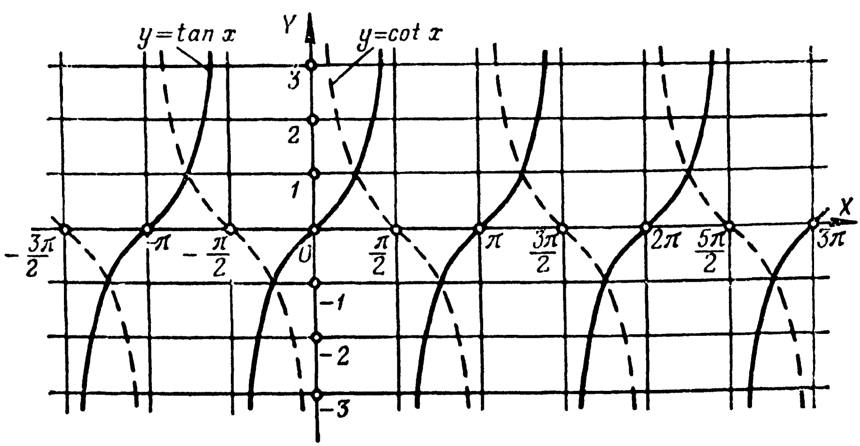
8*b* Semicubical parabola,

$y^2 = x^3$ or $\begin{cases} x = t^2, \\ y = t^3. \end{cases}$

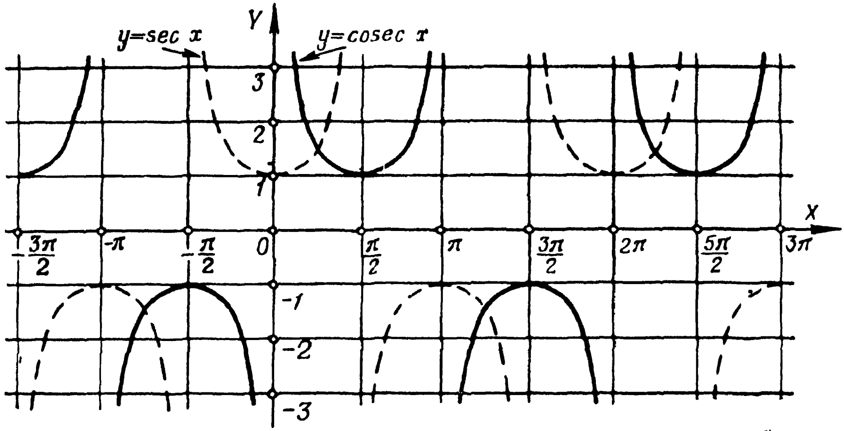




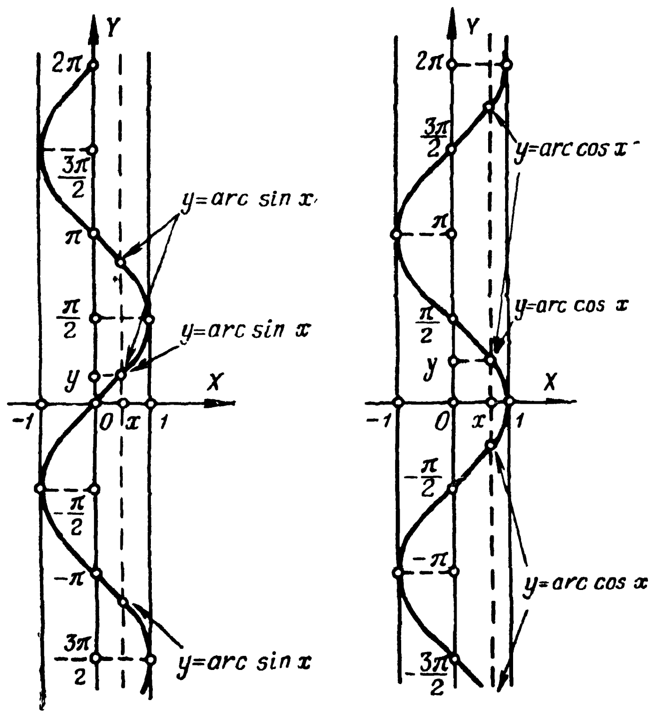
9. Sine curve and cosine curve,
$y = \sin x$ and $y = \cos x$.



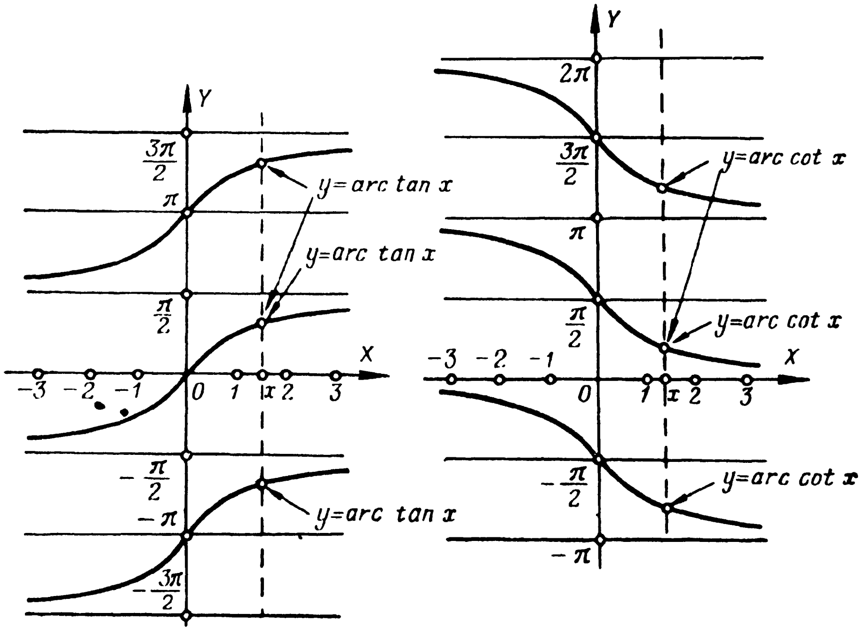
10. Tangent curve and cotangent curve,
$y = \tan x$ and $y = \cot x$.



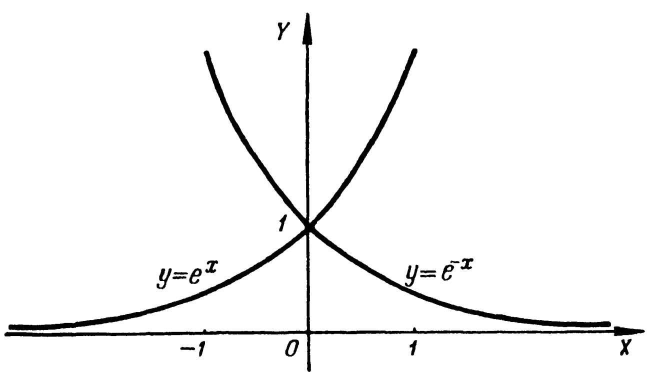
11. Graphs of the functions
$y = \sec x$ and $y = \operatorname{cosec} x$.



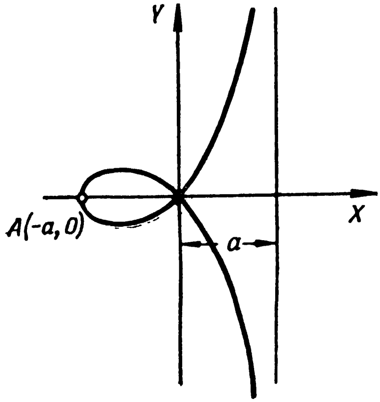
12. Graphs of the inverse trigonometric functions
$y = \arcsin x$ and $y = \arccos x$.



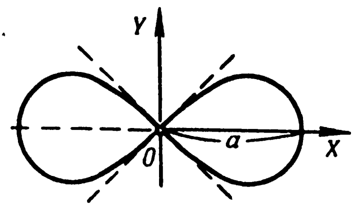
13. Graphs of the inverse trigonometric functions $y = \text{arc tan}\, x$ and $y = \text{arc cot}\, x$.



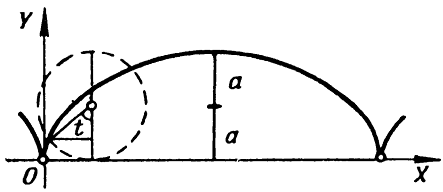
14. Graphs of the exponential functions $y = e^x$ and $y = e^{-x}$.



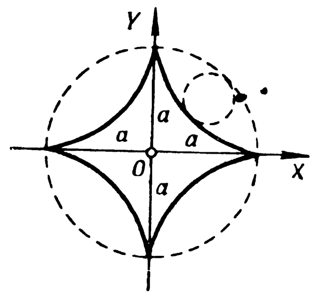
24. Strophoid,
$$y^2 = x^2 \frac{a+x}{a-x}.$$



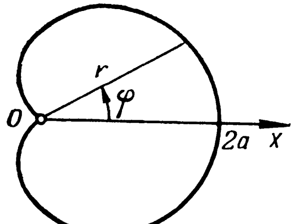
25. Bernoulli's lemniscate,
$$(x^2+y^2)^2 = a^2(x^2-y^2)$$
or $r^2 = a^2 \cos 2\varphi$.



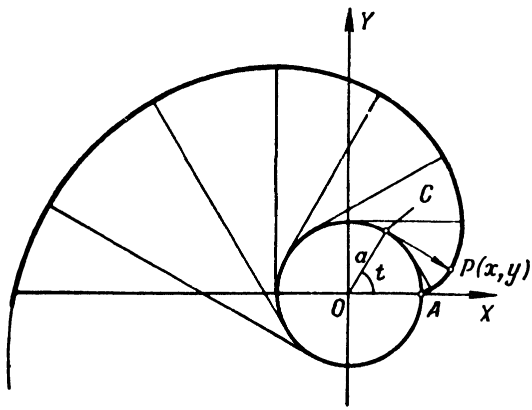
26. Cycloid,
$$\begin{cases} x = a(t - \sin t), \\ y = a(1 - \cos t). \end{cases}$$



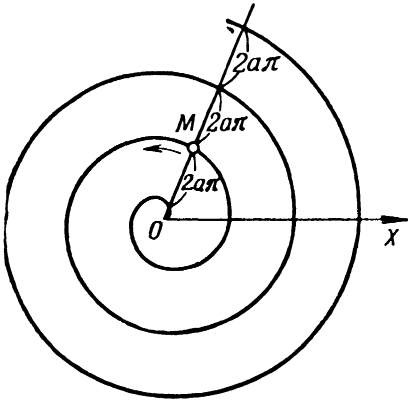
27. Hypocycloid (astroid),
$$\begin{cases} x = a\cos^3 t, \\ y = a\sin^3 t \end{cases}$$
or $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.



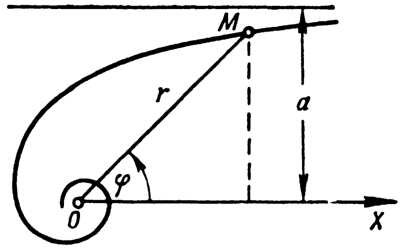
28. Cardioid,
$r = a(1 + \cos \varphi)$.



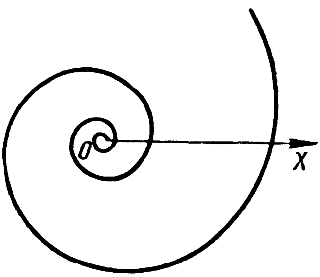
29. Evolvent (involute) of the circle
$$\begin{cases} x = a(\cos t + t\sin t), \\ y = a(\sin t - t\cos t). \end{cases}$$



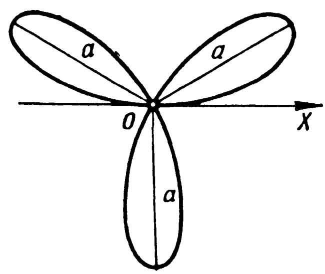
30. Spiral of Archimedes,
$r = a\varphi$.



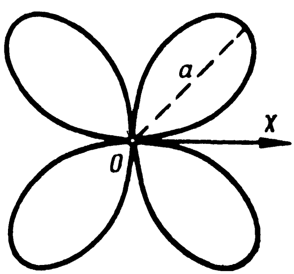
31. Hyperbolic spiral,
$r = \frac{a}{\varphi}$.



32. Logarithmic spiral,
$r = e^{a\varphi}$.



33. Three-leafed rose,
$r = a \sin 3\varphi$.



34. Four-leafed rose,
$r = a \sin 2\varphi$.

INDEX