Consequence 5. If $f, g : [a, b] \to R$ are simultaneously ascending or descending functions integrable in the inteval of $[a, b]$, $(f \neq 0)$ then:

$$\int_a^b f^m(x)g(x)dx \int_a^b f^{m-1}(x)dx \geq \int_a^b f^m(x)dx \int_a^b f^{m-1}(x)g(x)dx,$$

$(m \in N^*)$

If in Theorem 2. $h(x) = l(x) = const. > 0$ then we find the following proposition:

Consequence 6. If $f, g : [a, b] \to R$ are simultaneously ascending or descending functions integrable in the interval of $[a, b]$ then:

$$(b - a) \int_a^b f(x)g(x)dx \geq \int_a^b f(x)dx \int_a^b g(x)dx$$

which is no less then Tchebishev's inequality for integrals.

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1.19 Cubic Numbers, Which Are Tetrahedron Numbers At The Same Time

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This paper is the continuation of [1] and [2], where we presented the so called "figurative numbers", which where already studied by the disciples of Pythagora. Among others, we realised that in the case of m, n, p, q pozitive whole numbers:

since $a, \alpha, \beta \in N^*\setminus\{1\}$, on the basis of the equations (*) or (**), the equations (3.1) and (3.2) have no solution. So, the (III.1-III.2) system of equation has no solution .
IV. In the case of the equations $x = a^3$ and $x^2 - 1 = 6b^3, x = 2t + 1$, must be an odd number ($t \in N^*$), so $x^2 - 1 = 6b^3 \Rightarrow 2t(t+1) = 3b^3$, where $b = 2z$ must be an even number ($z \in N^*$), this
$2t(t+1) = 3b^3 \Rightarrow t(t+1) = 12z^3$. But t and $t+1$ are relative prime numbers, therefore there are such $\alpha, \beta \in N^*, \alpha \cdot \beta = b$ numbers, for which one of the following four cases is possible:
1). $t = 4\alpha^3$ and $t + 1 = 3\beta^3$, but $x = a^3$, on the basis of $x = 2t + 1$ and $t + 4\alpha^3$, $a^2 - (2\alpha)^3 = 1$, which is impossible, since $a - 2\alpha = \pm 1$ and $a^2 + a \cdot 2\alpha + 4\alpha^2 = \pm 1$ is not possible ($a \in N^*\setminus\{1\}, \alpha \in N^*$).
2). $t = 3\alpha^3$ and $t + 1 = 4\beta^3$, but $x = a^3$ on the basis of $x = 2t + 1$ and $t + 1 = 4\beta^3, (2\beta)^3 - a^3 = 1$, which similary to the previous one is impossible (In both cases we use the formula
$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$).
3). $t = 12\alpha^3$ and $t + 1 = \beta^3$, from where, on the basis of the equation $2t + 1 = a^3$, we obtain the equation
(4.1). $a^3 + 1 = 2\beta^3$
4). $t = \alpha^3$ and $t + 1 = 12\beta^3$, from where, on the basis of the first equation $2t + 1 = x = a^3$, we obtain the equation
(4.2). $a^3 - 1 = 2\alpha^3$
Since $a, \alpha, \beta \in N^*\setminus\{1\}$, on the basis of the equation (*), the equation (4.1) and (4.2) have no solution. Therefore the system, of equations (IV.1-IV.2) has no solution. This is why the only cubic tetrahedron number is $a = 1$, for which $1^3 = \frac{1(1+1)(1+2)}{6}$ (we obtained this, from equation (i) where $p = q = 1$). Though the result of the problem (P.2) may be a little bit surprising in comparison with the result of the analoque problem (P.2), the probable lack of analoque result was caused by the important differences between the solving of the quadratic and cubic diophantine equations. That is to say, that the solution of the problem (P.1) was given by the solution to the equation $a^2 - 2b^2 = 1$, while the solution to problem (P.2) is only analoque in form, with the previous equation $u^3 - 2v^3 = \pm 1$ (*), but the two types of equations are essentially different from point of field-arithmetic.

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1.20 The Sum Of The Relatively Prime Numbers With A Given Number

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Let $p_1^{\alpha_1}p_2^{\alpha_2}...p_t^{\alpha_t}$ the canonical form of n and let $r_1, r_2, ..., r_m$ the natural numbers smaller than n and relatively prime with n and let $\varphi_v(n) = n^v\left(1 - \frac{1}{p_1^v}\right)\left(1 - \frac{1}{p_1^v}\right)...\left(1 - \frac{1}{p_t^v}\right)$, where $\varphi_1(n) = \varphi(n)$ the Euler's totient function, and $\varphi_2(n) = \varphi(n)\psi(n)$, where $\psi(n)$ is the Dedekind's functions. Let be $\sum\limits_{k=1}^{n} k^i = a_0n^{i+1} + a_1n^i + a_2n^{i-1} + ... + a_in.$

Theorem. $r_1^i + r_2^i + ... + r_m^i = \frac{n^i m}{i+1} + (-1)^t a_2 n^{i-2} m p_1 p_2 ... p_t +$
$(-1)^t \sum\limits_{v=3}^{i} n^{i-2v+2} a_v \varphi_{v-1}(n) p_1^{v-1} p_2^{v-1} ... p_t^{v-1}$

Proof. We determine the expression $N^* = r_1^i + r_2^i + ... + r_m^i$ using Dirichlet's theorem, that is we add from 1 to n the powre i of the natural numbers from which we subtract the power i of those numbers that have a common divisor with n. If k is a divisor of n, then let $N(k)$ the sum of the powers i of the natural numbers smaller than n and divisible by k. Then:
$N^* = N(1) - N(p_1) - N(p_2) - ... - N(p_t) + N(p_1p_2) + ... + N(p_{t-1}p_t) - N(p_1p_2p_3) - ... - N(p_{t-2}p_{t-1}p_t) + ... + (-1)^t N(p_1p_2...p_t)$, but